Questioning for Understanding

- ∞ Can you explain this to me?
- ∞ What led you to this answer?
- ∞ Can you tell me more about that?
- ∞ What did you start with?
- ∞ What do these represent?
- ∞ What were you thinking here?
- ∞ How do you know?
- ∞ How did these/this help you understand what to do next?
- ∞ Is there another way you could try?
- ∞ What strategy are you using?
- ∞ How would you explain this to someone else?
- ∞ Does that make sense?
- ∞ Does that always work? Is it true for all cases?
- ∞ Do you see a pattern? Can you predict the next?

Important Ideas for Classroom Culture:

• Each and every student needs to feel that they belong and are important to the outcomes. Every student needs the class and the class needs every student. "None of us are as smart as we all are together"

• Ensure that students have a real chance to become engaged in and learn from the academic core they encounter.

• Children should experience the class as a community. They are expected and encouraged to contribute to the life of the whole group. Everyone needs help, everyone can help.

• Create an environment marked by the atmosphere of trust and respect between teacher and student and between student and student. Active listening is very important for everyone in the classroom.

• <u>All</u> responses are valued.

• Questioning doesn't stop when the expected response is offered. Question all responses, correct & mistakes. Let students know you're expecting them to explain and justify in addition to answering.

• No responses are "judged". Refrain from teacher comments / evaluations such as "very good", "correct", "wonderful", etc. You might respond: "You thought very hard about that." "I understand what you are saying."

• Celebrate each other's thinking. Display student work, students should compliment each other, applaud.

• Conversations are a special type of interaction in which both listening and participating are required. Ask students to compare 2 approaches/methods. Same? Different? How? Where do you see ... in the second strategy? Can't just listen without participating.

• Developing social competence includes turn-taking and negotiating skills, approach strategies, and a wide variety of communication skills.

• Social competence develops in the course of interactions with others. Children benefit from being engaged in more active and expressive processes than in passive and receptive ones. "*Never say anything a child can say*"

• Need a combination of individual time, small group & whole class. It is always important to follow up/sum-up/reflect with whole group sharing.

• Things will not always go smoothly or as expected. Need willingness to try, be thoughtful, and reflective of what goes well, what doesn't, and where to go next.

• Students know what they need to be a successful learner. Ask them!
Turn the experience back onto the students and ask them to discuss the lesson. What worked well for your group? For you individually? What difficulties did you encounter? What would help you to do better next time? Overall, how well do you feel your group worked together? What were your group's strengths? What do we need to improve on as a class?

• Create a class poster/rubric/pictures: What does it look like to work hard? How can we be helpful to others? What does it look like to be responsible? What do you need to be a successful learner? What can you do to help your classmates? What will we see when we work together? What will we hear when we work together?

RE-TEACHING VS RE-ENGAGEMENT

RE-TEACHING

- Teach the unit again.
- Address basic skills that are missing.
- Do the same or similar problems over.
- Practice more to make sure students learn the procedures.
- Focus mostly on underachievers.
- Cognitive level is usually lower.

RE-ENGAGEMENT

- Revisit student thinking.
- Address conceptual understanding.
- Examine task from different perspective.
- Critique student approaches/solutions to make connections.
- The entire class is engaged in the math.
- Cognitive level is usually higher.

Making Sense:
Teaching and Learning Mathematics With Understanding

8/23/06 9:20 PM

Star Ideas for every mathematics classroom, teacher, and student

- **Classroom Culture:**
    - **Collaboration and communication are essential**
        - Doing mathematics is a collaborative activity. It depends on communication and social interaction
        - Students can solve problems and construct understandings working collaboratively that they would not be able to accomplish working alone.
    - **Students should develop their own solution methods.**
        - A student's responsibility does not end when she or he has used a method successfully. The student must then work out a way to present and explain the method so others understand.
        - "I can only claim to understand a method if I can help others understand it."
    - **Mistakes are sites for learning!**
        - Making mistakes is a natural part of the mathematics process; it even may be essential sometimes. Mistakes simply are outcomes of methods that need to be improved.
        - Mistakes, if treated appropriately, can contribute to everyone's understanding.
    - **Correctness is determined by the logic of mathematics.**
        - Correctness of methods and solutions can and should be determined by the logic of the subject, rather than the teacher or a popular student.
        - Students must learn to live with some uncertainty as they evaluate the mathematical sense of a proposed method and solution.
        - Experiencing this kind of uncertainty and even learning to enjoy it is an essential part of thoughtful problem solving.

- **<u>The Nature of Classroom Tasks</u>:**
  - **Why are tasks important?**
    - Students learn from the kind of work they do during class, and the tasks they are asked to compete determine the kind of work they do.
    - Practicing paper-and-pencil skills on worksheets = faster at executing these skills = calculators
    - Watching the teacher demonstrate methods for solving problems = better imitators = monkey see monkey do
    - Reflecting on the way things work, how ideas and procedures compare of contrast, building relationships = ability to construct new understandings = able to think for themselves & outside of the box (or bubble)
    - <u>Tasks make all the difference!</u>
  - **Tasks should leave behind important residue.**
    - Too long we have been designing curriculum and instruction on the idea that we should first teach skills and then have students apply them to solve problems
    - It is better to *begin* with problems, allow students to develop methods for solving them, and recognize that what students take away form this experience is what they have learned.
    - Teachers can point out relationships, but they become meaningful as students use them for solving problems.
    - <u>The methods students first develop may not be the most efficient ones, but they will be methods students understand</u>.

- <u>**The Role of the Teacher**</u>**:**
  - **The most important role for a teacher becomes creating a classroom in which all students can reflect on mathematics and communicate their thoughts and actions.**
    - Teachers need to select sequences of tasks so that, over time, students' experiences add up to something important.
    - In traditional systems of instruction, teachers often describe their mathematical goals by listing skills and concepts they plan to teach.
    - In contrast, a curriculum that is filled with interesting problems and large, real life tasks, create goals that include engaging students in math and problem solving. The goals are not lists of skills and concepts, but to develop a greater understanding of mathematics.
  - **Teachers should not determine correctness exclusively.**
    - The final word on the correctness of a problem or task is provided by the logic of the subject matter and the students' explanations and justification that are built on this logic.
    - <u>The answer is in the math!</u>
  - **Teachers make the system work.**
    - Teachers play an active role and are responsible for guiding the mathematical activities of students and for establishing classroom culture that provides for reflection and communication.

TALK MOVES

Revoicing ("So you're saying that…)

Asking students to restate someone else's reasoning (Can you repeat what he just said in you own words?)

Repeating (Can you say that again?)

Asking students to apply their own reasoning to someone else's reasoning (Do you agree or disagree and why?")

Prompting students for further participation ("Would someone like to add on?")

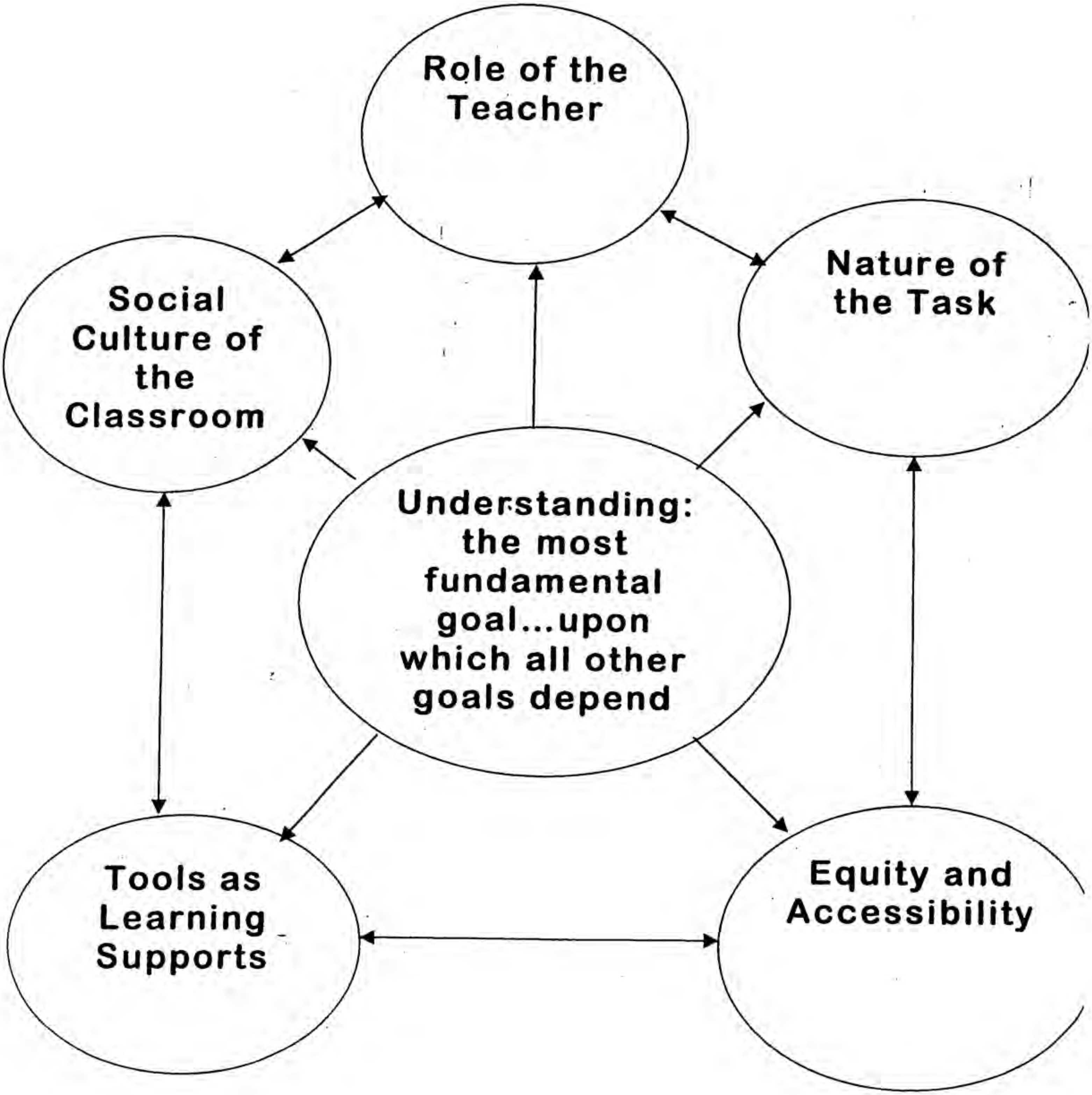
Using wait time ("Take your time…we'll wait…")

Teaching and Learning Mathematics with Understanding

Norms for a Healthy Social Culture

- Ideas and methods are valued.
- Students choose and share their methods.
- Mistakes are learning sites for everyone.
- Correctness resides in the logic of the mathematics.

Making Sense: Teaching and Learning Mathematics with Understanding by Hiebert, Carpenter, et al, Heinemann Press, c.1997


Role of the Teacher
Social Culture of the Classroom
Nature of the Task
Understanding: the most fundamental goal...upon which all other goals depend
Tools as Learning Supports
Equity and Accessibility

<table>
<tr><th colspan="4">Levels of Math Talk Learning Community: Teacher and Student Action Trajectories</th></tr>
<tr><th colspan="4">Components of the Math Talk Learning Community</th></tr>
<tr><th>A. Questioning</th><th>B. Explaining math thinking</th><th>C. Source of math ideas</th><th>D. Responsibility for learning</th></tr>
<tr><td colspan="4">Overview of shift over Levels 0–3: The classroom community grows to support students acting in central or leading roles, and shifts from a focus on answers to a focus on mathematical thinking.</td></tr>
<tr><td>There is a shift from the teacher as questioner to the students and teacher as questioners.</td><td>The students increasingly explain and articulate their math ideas.</td><td>There is a shift from the teacher as the source of all math ideas to students' ideas also influencing the direction of lessons.</td><td>The students increasingly take responsibility for learning and evaluation of others and of themselves. Math sense becomes the criterion for evaluation.</td></tr>
<tr><td colspan="4">Level 0: This is a traditional teacher-directed classroom with brief answer responses from students.</td></tr>
<tr><td colspan="4">Level 1: The teacher is beginning to pursue student mathematical thinking. The teacher plays a central role in the Math Talk community.</td></tr>
<tr><td colspan="4">Level 2: The teacher models and helps students build new roles. Some co-teaching and co-learning begins as student-to-student talk increases. The teacher physically moves to the side or back of the room and directs from there.</td></tr>
<tr><td colspan="4">Level 3: The teacher is a co-teacher and co-learner. The teacher monitors all that occurs and is still fully engaged. The teacher is ready to assist, but now in a more peripheral and monitoring role (coach and assister).</td></tr>
<tr><td>The teacher expects students to ask one another questions about their work. The teacher's questions still may guide the discourse.

Student-to-student talk is student-initiated, not dependent on the teacher. Students ask questions of each other and listen to responses. Many questions are "Why?" questions that require justification from the person answering. Students repeat their own or other students' questions until they are satisfied with the answers.</td><td>The teacher follows along closely to student descriptions of their thinking, encouraging students to make their explanations more complete; he or she may ask probing questions to make explanations more complete. The teacher stimulates students to think more deeply about strategies.

The students describe more complete strategies; they defend and justify their answers with little prompting from the teacher. The students realize that other students will ask them questions, so they are motivated and careful to be thorough. Other students provide support with active listening.</td><td>The teacher allows for contributions from students during his or her explanations; he or she lets the students explain and "own" new strategies. The teacher is still engaged and deciding what is important to continue exploring. The teacher uses student ideas and methods as the basis for lessons or mini-extensions.

The students contribute their ideas as the teacher or other students are teaching, confident that their ideas are valued. The students spontaneously compare and contrast and build on ideas. Student ideas form part of the content of many math lessons.</td><td>The teacher expects students to be responsible for co-evaluation of everyone's work and thinking. He or she supports students as they help one another sort out misconceptions. He or she helps and/or follows up when needed.

The students listen to understand, then initiate clarifying other students' work and ideas for themselves and for others during whole-class discussions as well as in small group and pair work. The students assist each other in understanding and correcting errors.</td></tr>
</table>

Table 1

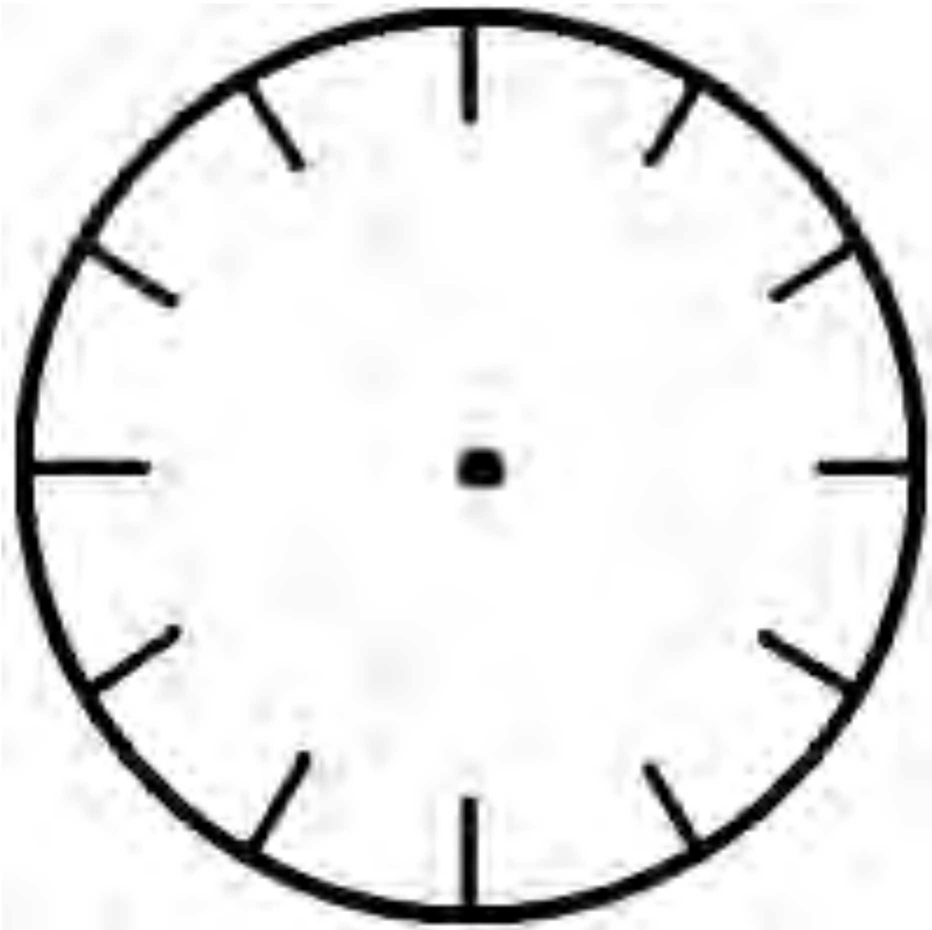
Monday:	Tuesday:	Wednesday:	Thursday:	Friday:
- K ice breaker activity - Adult # Talk - CCSS #3 Justify/Argue - Debrief #talk through learner lens - Video Katie #talk - Start Reading: Building Community in the Classroom	- Adult # Talk (number of the day – with video - M: Debrief grade level ideas - CGI Verbage of CCSS Number & Operations	1st Site Visit - Dot Talk - Student Task	- Adult # Talk - 'What is Re-engagement? - How will we use Re-Engagement at Site Visit?	- Adult # Talk - Debrief visit: Number talks & Re-engagement - Plan classroom # talks for September (bring student # talk work to Sept. PD)
- Finish reading - JEMS – small group activity - K's Classroom Culture w/ handout - Small groups – share their ideas/strategies - Comment Cards	- Problem Solving Classroom - Organization Tips - CCSS #1 Making Sense of Problem & Persevering - Comment Cards	- Debrief Site Visit - Plan # Talks - Bar Models - Comment Cards	2nd Site Visit - Small group # Talks - Re-Engagement Lesson - Comment Cards	- Assessment Tasks as used for Re-Engagement - Mathy Activities (Marilyn Burns, etc.) - Small Groups share/teach others "What's your fave math activity for beginning of the year?" - Comment Cards

Changing our approach and how we are teaching Mathematics rather than focusing on what we use to teach math.

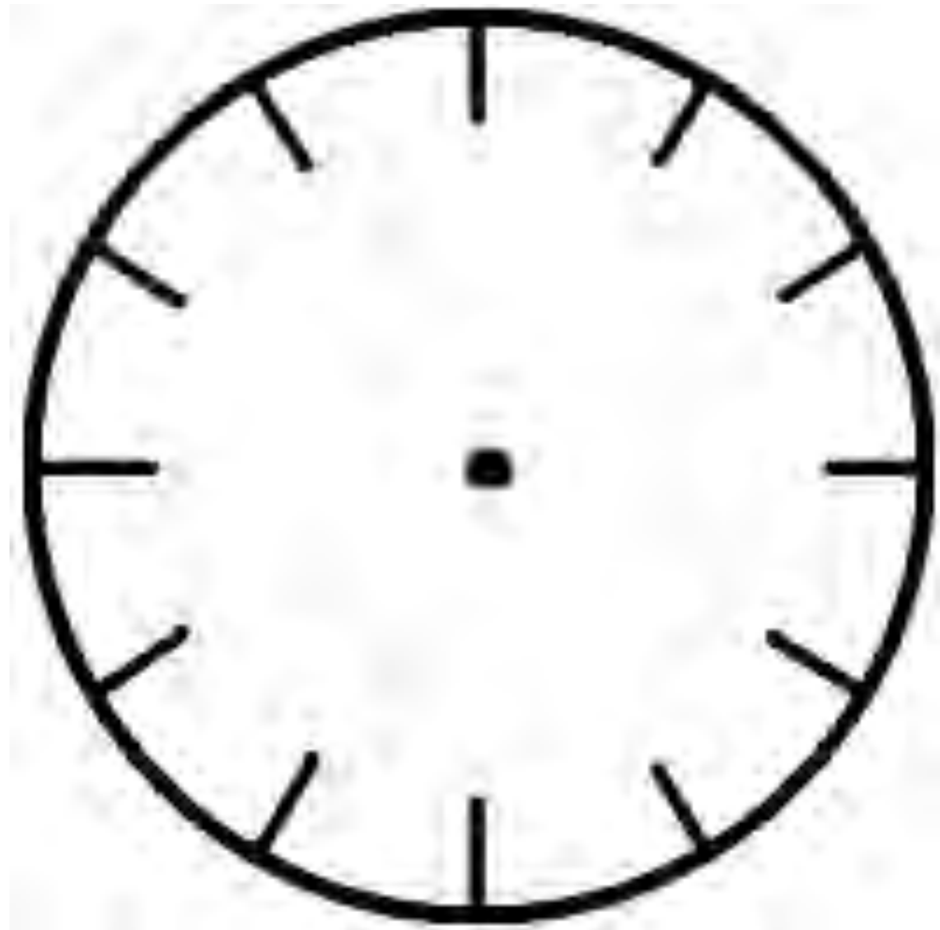
K to copy: Building Community in the Classroom, Blank calendar for # talk planning, classroom culture info, Re-Engagement vs. Re Teach poster

M to copy: CGI Matrix, Bar Model stuff, student # talks, CCSS Math Practices info used in Chicago

My Clock Partners:



My Clock Partners:



Month:____________________ Number Talk Focus:____

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday

Icebreakers
Group Profile
Materials: newsprint, markers, tape
Preparation: Trace an outline of the human body on newsprint. List the following topics outside the outline next to the coordinating body part:
Head: dreams or goals we have (for our community)
Ears: things we like to listen to
Eyes: How we like other people to see us
Shoulders: problems young people may have to face.
Hands: things we like to make or do (with our hands)
Stomach: things we like to eat
Heart: things we feel strongly about
Right foot: places we would like to go
Directions: Post outline of body on the wall. Invite participants to come up to the poster and write things or pictures to represent each area for them. This is done grafitti style, free form.
After everyone has had a chance to participate, ask for volunteers to report to the group on what is listed.
Discuss:

What are common interests? Shared goals? Dreams? Were there any themes? What are the things we feel strongly about? How do these relate to our group's work?

Incorporation
Explain that this game is about forming and reforming groups as quickly as possible. Don't worry if you are not even into the first group by the time the next group is called, just head to the next group. The idea is to meet many different groups of people as fast as possible. Get into a group of three...go!

Other suggestions:
A group of five with everyone having the same color eyes as you.
With the same last digit in their phone number as yours.
Wearing the same size shoe as you.
Get into a group of three people and make the letter "H" with your bodies.
Find everyone else born in the same month as you
Think of the first vowel in your first name, find four with the same vowel.

Sign Up Here

Materials: 6-10 pieces of large newsprint, tape, and pencils.

Preparation: Put pieces of the newsprint around the room. From the list of topics below, write a different topic of interest on the top of each newsprint. Also include a related question you want people to answer about each topic. (Topics can vary according to the age and interests of group involved):

I like to speak or perform in public. (What group(s) have you spoken to or performed in front of?)

I like to work on computers. (What programs do you know?)

I can speak a language other than English. (Which?)

I would be excited to travel in the U. S. or abroad. (Where? Where have you been?)

Making friends is an important part of my life. (Who are your best friends?)

My family is one of the things that makes me happy. (Something I like about them?)

There are things that I would like to change in this school. (What?)

There are things that I would like to change in our community. (What?)

The voting age should be moved from 18 to 21. (If you could vote, what law would vote to change?) I have organized or helped to organize an event, celebration, fund-raiser, meeting, wedding, or conference. (Describe.)

Instruct participants to walk around the room, look at the different topics and sign their name on any of the sheets that represent topics in which they have an interest, and to make a comment answering the question on each sheet.

After everyone has had a chance to sign the sheets, ask one person that has signed each sheet to read the names of the people that have signed that sheet and any comments.

Discussion: What interests does the group have? How many different interests are represented in the group? Which chart had the greatest interest? Which chart had the least interest? What does this say about the group as a whole? Is there a pattern? What comments are made?

Synthesis: Explain how these skills are important for community organizing and how each of them will contribute their interests and skills making the group stronger.

Who Am I?
The leader tapes the name of a famous person on the back of each participant. (i.e. Fred Flintstone, Mary Lou Retton, Bill Clinton, etc.) The group member is not to see who is taped to their back. Their task is to find out who they are. The participants go around the room asking others only yes or no questions. If the member receives a "yes" answer, they can continue to ask that person questions until they receive a "no" answer. Then they must continue on to ask questions to someone else. When a group member figures out who they are, they take off the tag, put it on the front of their shirt, and write their own name on it. That person can then help others find out who they are. The exercise concludes when everyone has discovered who they are.
Variation: Use names of famous pairs (like Syskell and Ebert, Bert and Ernie) and do a partner activity after the game.

Energizers
Chalkboard Sentences
Tell participants they will be competing to see which team is the first to complete a group sentence. Next, divide participants into two teams. If the group contains an uneven number, one person may compete twice. The leader sets up black-boards or newsprint for each team. The teams then line up 10 feet from their board. After giving the first person in each team’s line a piece of chalk or marker, explain the rules of the game. The rules are: Each team member needs to add one word to the sentence. Payers take turns; after they go to the board and write one word, they run back to give the next player the marker, and then go to the end of the line. (The sentence must contain the same number of words as there are members on the team.) A player may not add a word between words that have already been written. After, discuss the value of anticipatory thinking and the importance of individual cooperating in a group task).

2. **Spelling List of Names** - My first spelling list is made up of the names of everyone in the class. To make a game out of learning the names, I give each person an index card and pair the students randomly. The students interview each other and take notes about their partner's interests. Then we write down everyone's name on a large spelling list. While I'm writing the person's name, their partner stands up and introduces the student to the class. Students copy the names down and we use them in other activities like Bingo. I generally have them learn only the spellings of the first names, and if the class is large I divide the test into 2 different parts.

3. **Classmates Mix** - This is a fun icebreaker activity. Students mix around the classroom until you say give a signal to stop. Then they pair up with the closest person. You call out an icebreaker topic such as the ones below. Students talk over their answers until you call time, and then they begin mixing again. Continue with several rounds for as long as time allows. You might try 3 rounds one day and 3 rounds the next day if your students have trouble handling the movement at first. (The STOP technique works well for classroom management during classbuilders.) Here are some discussion topics to get you started:

- Share a little information about yourself and your family.
- What are some of your favorite things? Talk over your favorite foods, colors, animals, or anything else that's a favorite of yours.
- What do you like to do in your free time?
- What's the best book you have ever read? What did you like about it?
- What's the best movie you have ever seen? Why did you like it?
- What's your favorite subject in school? What do you like about this subject?
- What are your strengths? What kinds of things do you do well?
- How would you change this school if you were the principal?
- What can students do to make school a better place to be?

The 512 Ants on Sullivan Street

A Lesson for Third Graders

by Maryann Wickett and Marilyn Burns

From Online Newsletter Issue Number 20, Winter 2005–2006

This lesson is excerpted from Maryann Wickett and Marilyn Burns's new book, Teaching Arithmetic: Lessons for Extending Place Value, Grade 3 *(Math Solutions Publications, 2005). Children's understanding of place value is key to their arithmetic success with larger numbers, and this book is important for fostering their understanding. In this lesson, based on a children's book, children think about what happens to the magnitude of numbers when they double over and over and how doubling relates to addition and multiplication. They also apply their understanding of place value to solve subtraction problems.*

The students gathered around me, chatting about the book I was holding, *The 512 Ants on Sullivan Street,* by Carol A. Losi (Scholastic, 1997). I settled them down and read aloud the book's first two spreads, which introduce one ant walking away with a crumb and two ants taking part of a plum. I asked the children to predict the number of ants that would be taking food from the picnic basket on the next page. Some students thought it would be three ants, stating that there could be a pattern that increased by one ant each time. Others thought there would be four ants, suggesting a doubling pattern. A few made random guesses, thinking that there was no pattern at all to the number of ants.

I continued reading and we learned that next, four ants make off with a barbequed chip. I recorded on the board:

Number of Ants

1

2

4

As I finished recording, students waved their hands, eager to share. I called on Jessie.

Jessie said, "It's a doubling pattern. One plus one equals two. Two is next. Two plus two equals four. And four is the next number. I think the next number after that will be eight because four plus four equals eight."

I added Jessie's idea to the list:

Number of Ants

1

2 = 1 + 1

4 = 2 + 2

I read the next spread to verify that eight ants come along next, this time carrying off a bacon strip. I said, "It seems as if there is a doubling pattern. What's a way we can show a doubling pattern with multiplication?"

"Doubling is like timesing by two," Karena replied.

To be sure Karena understood the connection between doubling and multiplying by two, I nudged, "It would be helpful if you could explain your thinking and give an example."

Karena explained, "The times sign means groups of. And doubling means two of something. So instead of one plus one, you could think of it as two groups of one. You'd write that with a two, then a times sign, and then a one. That equals two just like one plus one equals two."

I pointed to 2 + 2 = 4 and asked the students, "How would I write this as a multiplication sentence?"

Christopher replied, "Two times two equals four. There are two twos, so that's two groups of two or two, two times."

With the students' help, the recording soon looked as follows:

1

$2 = 1 + 1 = 2 \times 1$

$4 = 2 + 2 = 2 \times 2$

$8 = 4 + 4 = 2 \times 4$

The children continued to make predictions about subsequent numbers of ants. I verified by reading the story and continued to record on the board. When we got to thirty-two ants, I asked, "If I wanted to put the ants into groups of ten, could I? Would I have any ants left over?"

Karlee said, "There will be ants left over, but I'm not sure why."

I said, "Put your thumb up if you agree that there will be ants left over if we put thirty-two ants into groups of ten. Put your thumb sideways if you're not sure, and put your thumb down if you think there will be no ants left over." All thumbs were up. I said, "Raise your hand if you'd like to explain why there will be leftover ants."

Olina shared, "If there's exactly a group of ten, the number ends in zero. Like ten has one group of ten and ends in zero. Twenty has two groups of ten and ends in zero. Thirty-two doesn't end in zero. It has three groups of ten and two leftover ants."

Tobias said, "You can use multiplication. Three times ten equals thirty and four times ten equals forty. Thirty-two gets skipped. I agree with Olina; there will be two ants left."

I continued reading the story and together we finished the chart we'd started showing the patterns of doubling both by adding and by multiplying by two.

Number of Ants

1

$2 = 1 + 1 = 2 \times 1$

$4 = 2 + 2 = 2 \times 2$

$8 = 4 + 4 = 2 \times 4$

$16 = 8 + 8 = 2 \times 8$

$32 = 16 + 16 = 2 \times 16$

$64 = 32 + 32 = 2 \times 32$

$128 = 64 + 64 = 2 \times 64$

$256 = 128 + 128 = 2 \times 128$

$512 = 256 + 256 = 2 \times 256$

Subtraction with Regrouping

I then asked, "Remember in the story when thirty-two ants hauled a wing and a leg? What if they needed fifty ants for that job? How many more ants would they need?"

When most of the students had their hands up, I called on Adama. She said, "You need eighteen. The problem could be fifty minus thirty-two, but all you have to do is start with thirty-two and count up to fifty. Thirty-two plus ten equals forty-two; then eight ones make fifty. Fifty minus thirty-two equals eighteen."

I recorded on the board:

Adama $50 - 32 =$

$32 + 10 = 42$

$42 + 8 = 50$

$50 - 32 = 18$

Roberto said, "You could start with thirty-two and add eight ones to make forty and then one group of ten to make fifty. Eight and ten equal eighteen, so thirty-two plus eighteen equals fifty."

I recorded:

Roberto $32 + 8 = 40$

$40 + 10 = 50$

$10 + 8 = 18$

Tina had a different strategy, and I recorded hers as well.

A Second Subtraction Problem

I then wrote on the board: *At a nearby picnic, 64 ants want to carry away a huge dill pickle. Although the ants try their best, they can't move the pickle. They decide that 100 ants could do the job. How many more ants are needed?*

I handed each student a sheet of paper and I explained, "I'm interested in knowing what each of you thinks and understands, so please do your own work."

While the students worked, I circulated through the class, observing carefully. My goal was to gain a clearer understanding of what strategies the students were using and what misconceptions they might have had. I also was interested to see which students had access to the problem and which didn't, and which students would use their understanding of place value and which would rely on the standard algorithm or another means of figuring. I was also interested in which students would solve the problem using subtraction and which would choose addition.

Roberto wrote the problem as a subtraction problem: *100 – 64.* His written explanation stated, *I add to 64 to 100 and it took me 36 fingers. That's 3 tens and 6 ones. 36.*

Keara used subtraction and her knowledge of place value to solve the problem. Although she subtracted, she did not use the standard algorithm. She subtracted the tens first and then the ones. (See Figure 1.)

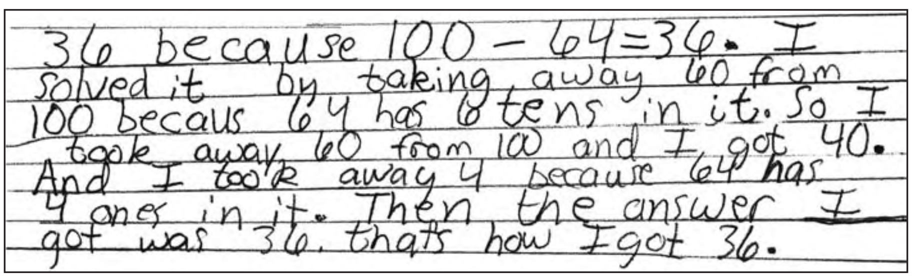

36 because 100 - 64=36. I solved it by taking away 60 from 100 becaus 64 has 6 tens in it. So I took away 60 from 100 and I got 40. And I took away 4 because 64 has 4 ones in it. Then the answer I got was 36. thats how I got 36.

Figure 1. Keara used her knowledge of place value to successfully solve the problem.

Adama first solved the problem using addition; then she used the standard algorithm as a check. (See Figure 2.)

Jessie counted on from sixty-four, using tally marks grouped in tens to help her find an answer. (See Figure 3.)

Tobias started with 64 and counted up by tens until he got to 104, for a total of four tens, or forty. Then he counted four back from 104 to get to 100. Finally, he subtracted four from the four tens, or forty, to get thirty-six. (See Figure 4.)

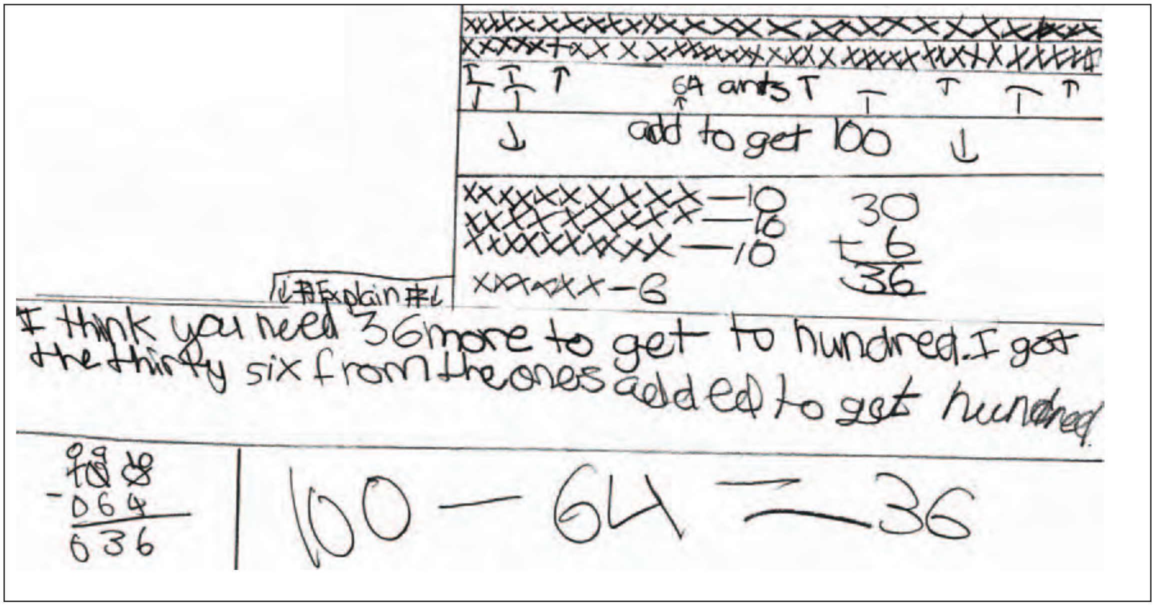


Figure 2. Adama counted on using tens and ones to find the solution.

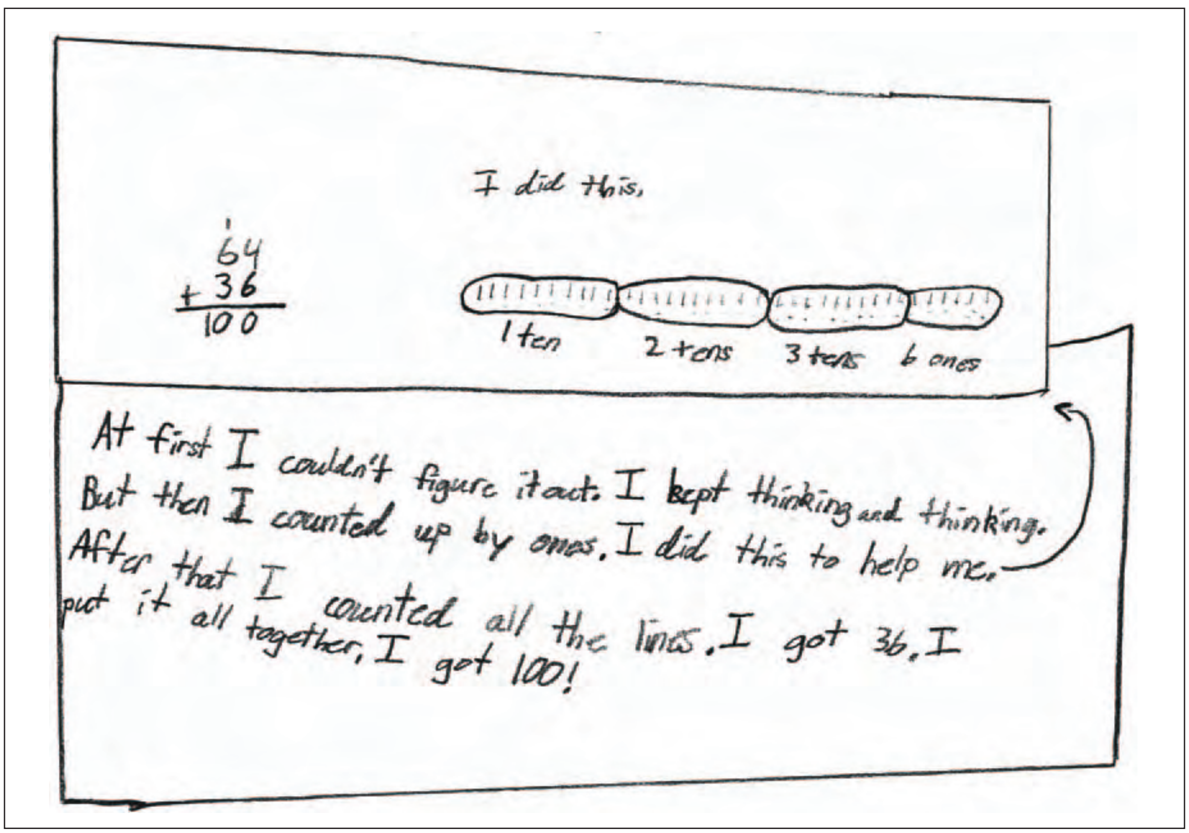


Figure 3. Jessie used tallies and added on groups of tens and ones to help her solve the problem.

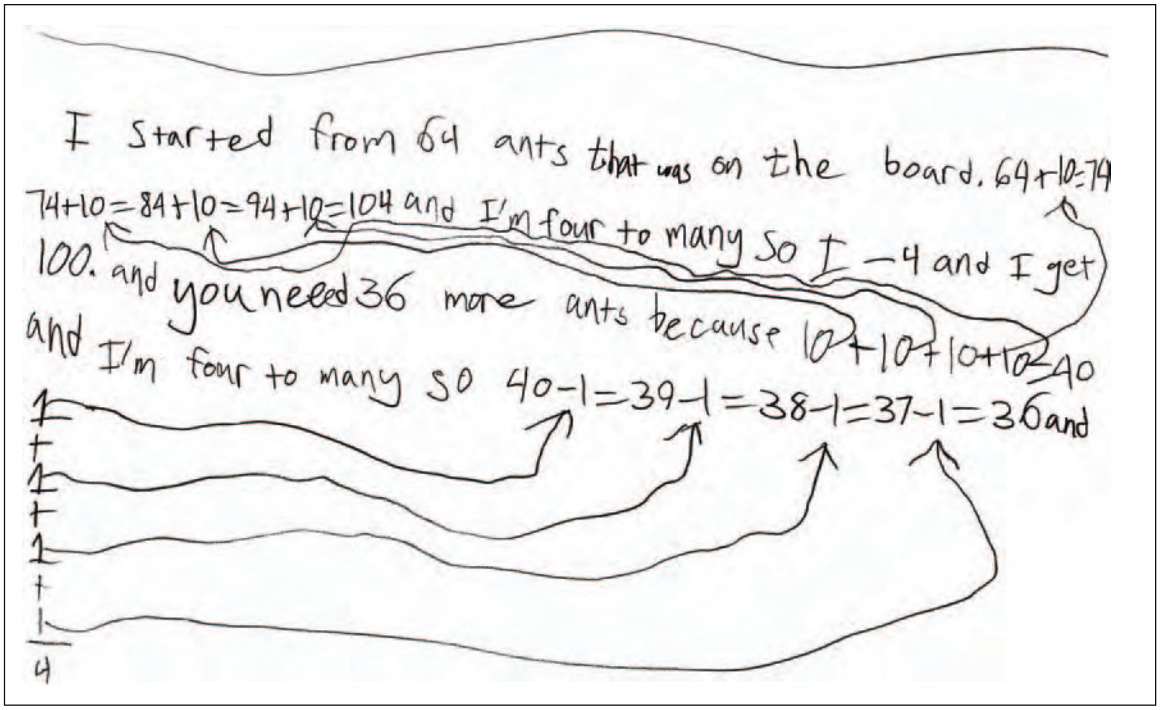


Figure 4. Tobias successfully solved the problem.

A few of the students needed help, and in order to give them assistance, I gave the others a third problem to solve. I wrote on the board:

> *512 ants have discovered a watermelon.*
>
> *1,000 ants are needed to carry it away.*
>
> *How many more ants are needed?*

I then gathered Jenae, Jade, and Rachel to give them help while the others went to work.

Behavior Reflections

Reasons for My Behavior

Name ______________________

Date ________________

Consequences of My Behavior

Description of My Behavior

How do I feel?

How has my behavior affected others?

Other Consequence(s)

Plan for Improvement ____________________________

____

____

____

____

Student ________________________

Teacher________________________

Parent ________________________

Math Solutions® PROFESSIONAL DEVELOPMENT

Big and Little

A Lesson for Third Graders

by Jamee Petersen

From Online Newsletter Issue Number 14, Summer 2004

Understanding the concept of scale is not easy for young children, but Steve Jenkins's book Big and Little *(Houghton Mifflin, 1996) can help. The illustrations in the book show animals at the same scale, making it easy to compare their sizes visually. In this lesson, Jamee Petersen uses the book to introduce third graders to the idea that a 1-inch illustration can represent 8 inches in the real world. Jamee's forthcoming book,* Math and Nonfiction, Grades K–2, *will be published by Math Solutions Publications in fall 2004; in her new book, she includes a lesson about how to use Big and Little with kindergarten children.*

When I read *Big and Little* to a class of third graders, I began with Steve Jenkins's note to his readers in which he explains that all of the animals in the book "are related but different in size." After explaining a bit more about this, Jenkins writes, "All of the creatures in this book are illustrated at the same scale (one inch equals eight inches) so animals throughout the book can be compared."

After reading the book through once, I said to the class, "You know, we have examples of big and little right here in this room. How many of you know how tall you are?"

Some students knew an exact measurement. Emma said, "I am three feet and ten inches." Others gave a relative measurement. Alison said, "I know I'm taller than Jack but shorter than Hannah."

"Remember the author's message at the beginning of *Big and Little?"* I asked the class. "Some of animals in this book are too big for the author to show as their actual size, but because he wanted us to be able to compare their sizes, he made a scale. The animals in this book are all drawn so that one inch in the drawing equals eight inches in the real world. That was his scale." On the board I wrote:

1 inch (in the book) = 8 inches (in the real world)

"What do you think this means?" I asked.

"It means that you times it by eight," Alan said.

"It means that if an animal is exactly eight inches tall, then it is one inch tall in this book," Carrie said proudly.

Max agreed but explained it in this way after getting a ruler: "It means that one inch in the book," and he covered up the ruler with his arm so that only 1 inch showed, "really equals eight inches." Max then moved his arm so that 8 inches showed.

While some students seemed to grasp this concept, I knew that others didn't. I suggested we "test" Max's idea on the illustration of the Siamese cat shown sitting on the book's first page. Knowing all the students would have a good idea of the size of a house cat, I took Max's ruler and carefully laid it next to the cat in the book.

A few students in the front helped read the measurement, explaining that the cat was more than 1 but less than 2 inches. “The cat is one and a half inches tall,” Aaron announced to the rest of the class.

“So if the illustration of the cat is one and a half inches tall and we know that one inch equals eight inches, then how tall is the cat in real life?” I asked.

To solve this problem, some students went to get rulers while others sat thinking quietly. After a few moments, I called them all to attention and said, “I’m interested in knowing how you thought about solving this problem.” Many hands went up.

Jose said, “Well, the cat is one and a half inches. One inch is eight inches and I know that half of eight is four, so the real cat is twelve inches tall.”

Kara said, “I knew my doubles. And halves and doubles are kind of the same thing, so I just said eight and then half of eight is four and eight plus four equals twelve. So the cat is twelve and that seems right.” Kara placed her hand about 12 inches from the floor and added, “This is about how high my cat is when he’s sitting.”

Some hands went down, indicating to me that those students had thought about the problem in a similar way. Still others were still waving wildly.

“I did it different,” Sara said. “I took the ruler and found eight because I knew the cat was one whole inch and one inch is eight. Then I looked at the ruler and I just saw that four was half of eight so I just moved my hand up like this,” Sara explained, as she deliberately moved her hand up the ruler, counting on four more from the 8. “So that’s when I saw it was twelve inches,” she concluded.

“Twelve inches is a foot, so the answer could be one foot tall,” surmised Justin.

“So we figured out how tall the cat in the book would be in real life; is there a way we could figure out how tall you would be if you were in this book?” I asked.

The students first looked puzzled but then began to smile as they anticipated the activity.

“Yeah, we would just need to figure how tall we are and then go back by eight,” Carrie said.

“Tell me more,” I asked.

“Well, it’s the same way we figure out the cat but in reverse,” Carrie added.

I asked Kathy to come to the front of the room to help me. She came up and stood in front of the others. “I would need to measure Kathy’s height first,” I said. First I had other children help by holding rulers, one on top of the other, but then I suggested that maybe another measuring tool would work better. Some wanted to use a yardstick, with Kathy lying down; others thought a measuring tape would be best.

I tried all of their suggestions, modeling many ways to measure and purposefully demonstrating common mistakes, such as not having the tape tight or leaving spaces between the rulers rather than having them touch. We found out that Kathy was just over 50 inches tall.

"OK, Kathy is about fifty inches tall," I said, rounding to the nearest inch. "How can we figure out how tall a drawing of Kathy should be if we used the scale of one inch equals eight inches?" I gestured to the board where I had earlier written *1 inch (in the book) = 8 inches (in the real world).*

I was surprised at how many hands shot up. "We would have to find out how many eights are in fifty," Jose offered.

"Yes, that's right," I said. At this point, I explained to the children that their task was to figure out how little they would be if they were illustrated in the book *Big and Little.* "You may use any tools you think would help you. You may work in teams to help each other get an accurate measurement, but I'd like each of you to have a written record of what you did. Your paper should help others understand your thinking. Your explanation may have words, numbers, and a picture—whatever you think will help explain your thinking." I placed some white drawing paper out for the students to record their thinking and they got to work.

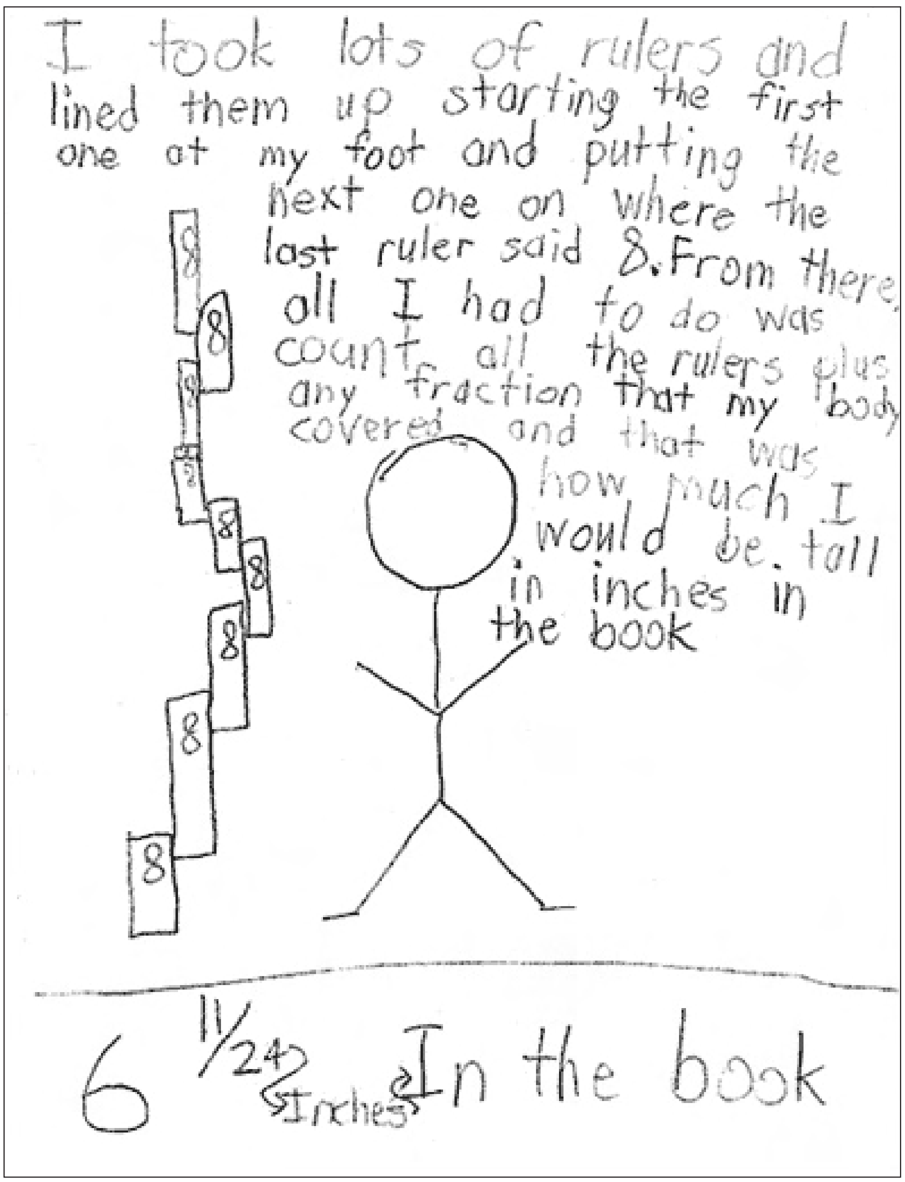


Figure 1. Nathan used a ruler to help solve the problem.

Nathan decided working with segments of 8 inches would be easiest. I understood how he came up with the 6 in his answer but the 11/24 puzzled me. When I asked Nathan about his answer, he explained, "I got eleven–twenty-fourths because there were six rulers and then there was five and a half of the next one; five and a half of twelve inches, so I added on eleven–twenty-fourths. So I had six and eleven–twenty-fourths." Nathan's reasoning wasn't accurate, but I was impressed that a student his age was comfortable enough in his number sense and reasoning skills to attempt converting a mixed number into a fraction.

When I looked at Alex's work, I could see he had erased 53 and changed it to 52. I asked him to explain. He said, "I went down to fifty-two because it's an even number and I usually work better with even numbers."

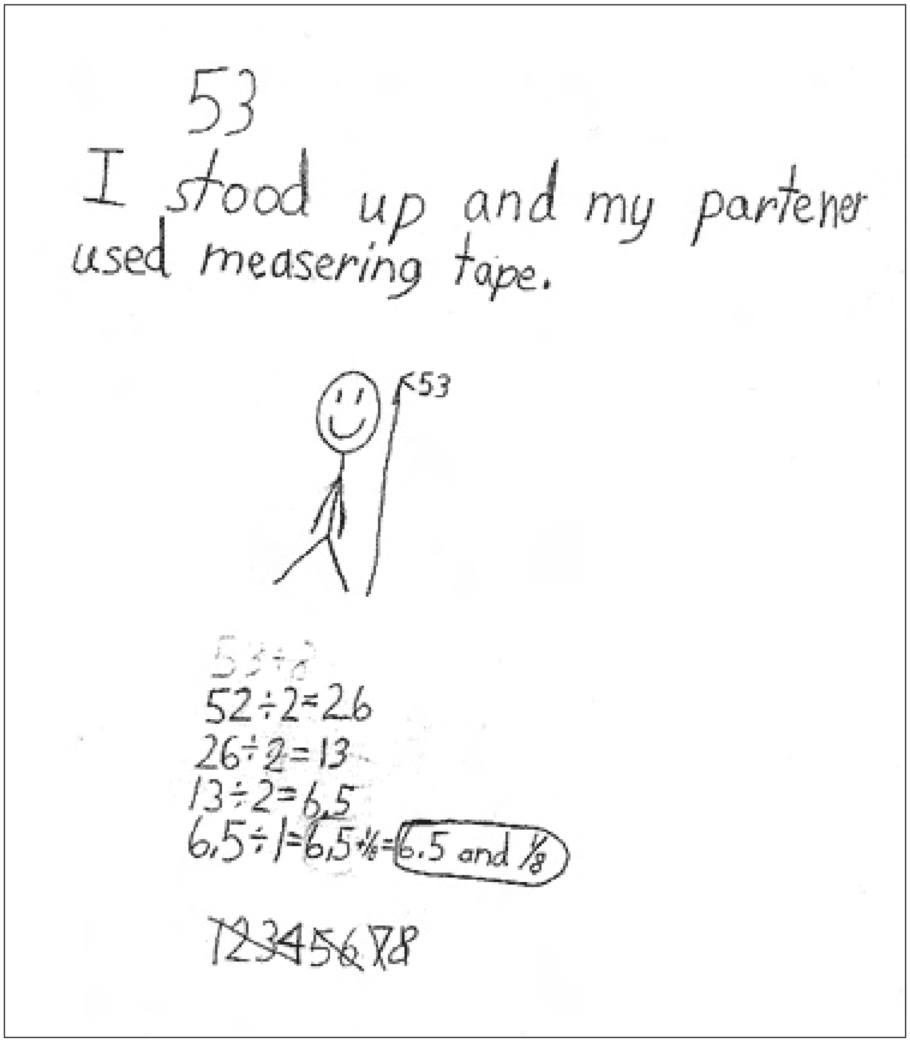


Figure 2. Alex's work with division.

Molly's and Marie's papers were similar. Both girls grouped by eight, one using number symbols and repeated addition and the other using tally marks in groups of eight.

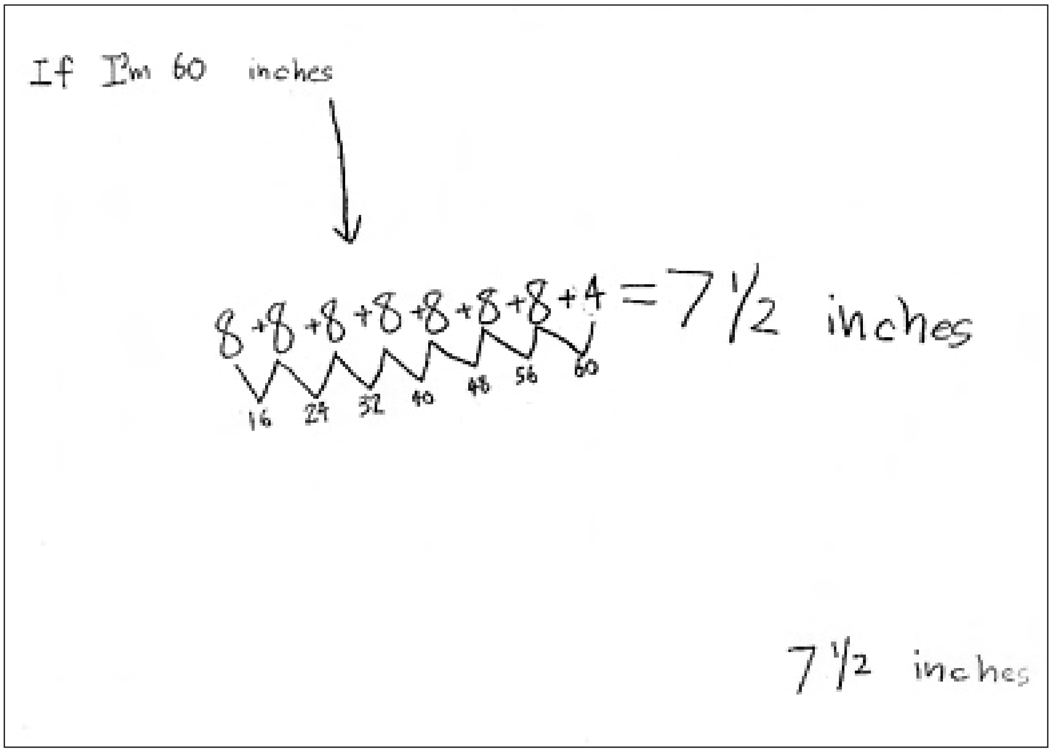


Figure 3. Molly recorded her work as repeated addition and then counted by eights.

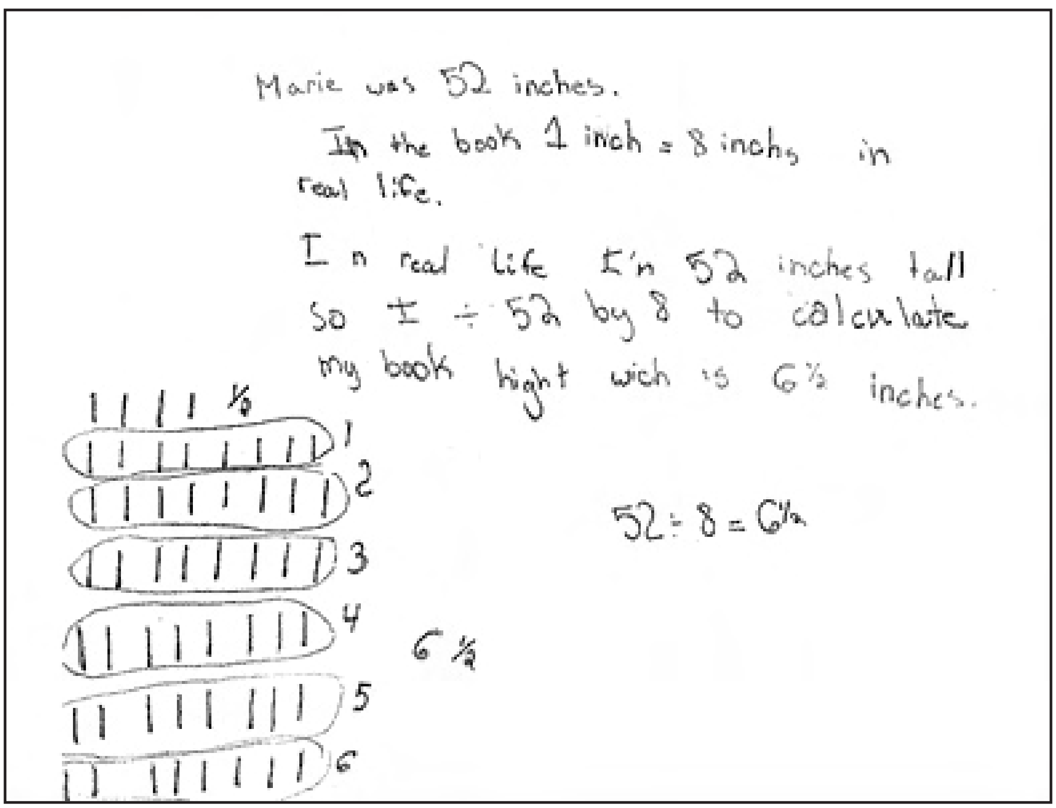


Figure 4. Marie used a grouping model.

The next day I read *Big and Little* once more to my students. Before reading, I asked them to use their work from the previous day to create simple drawings of themselves using their "book heights" based on the scale of 1 inch being equal to 8 inches. They cut out their drawings and brought them to the rug with them as they gathered for the story. As I read, students were able to compare their heights with the heights of animals included in the book. Placing their drawing cut out next to the big tiger and the little hummingbird brought an even greater depth of understanding of the different sizes of these animals.

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The Button Game

A Lesson for First Graders

by Vicki Bachman

From Online Newsletter Issue Number 11, Fall 2003

In First-Grade Math: A Month-to-Month Guide *(Math Solutions Publications, 2003), Vicki Bachman offers teachers a thoughtful, practical way to plan for a year of comprehensive and coherent math instruction. Each month is organized so that it focuses on particular number concepts and skills, addresses another of the content strands of the math curriculum, and presents ways to connect mathematics instruction to other areas of the curriculum.* The Button Game, *which appears in the "September" chapter, introduces students to the concepts of* same *and* different. First-Grade Math *is one of a three-book series that also includes* Second-Grade Math *and* Third-Grade Math.

Before introducing this activity, gather the following materials:

- 30 buttons
- zip-top bag to hold the buttons
- plate large enough to display the buttons
- copy of *The Button Box,* by Margarette Reid (Penguin Books, 1990), or *Frog and Toad Are Friends,* by Arnold Lobel (Harper Trophy, 1971)

Invite the children to join you in the circle area. Read either *The Button Box,* by Margarette Reid, or the story "The Lost Button" from *Frog and Toad Are Friends,* by Arnold Lobel. After reading the story, ask if anyone has some buttons on his or her clothing today. Invite a few children to show their buttons. Encourage the class to discuss colors, shapes, numbers of holes, and other attributes of these buttons. Observe that all of these buttons are alike in some ways and different in others.

Pour your bag of buttons onto a plate so that everyone can easily see them. Hold up a button and discuss the article of clothing that it might have been attached to. Hold up another button and call attention to the ways in which the buttons are alike and different.

Put all of the buttons back on the plate, then walk around the circle, asking each child to select a button and place it on the floor in front of him or her. Take a button for yourself and sit down with the children.

Place your button next to that of the child who is sitting to your left or right. Tell the class that you and your partner are going to find one thing about your buttons that is alike and one thing that is different.

Ask your partner what he or she notices about the two buttons. Use the responses to identify an attribute that is the same for both buttons, then discuss a way that the buttons are different. (For example, perhaps they are the same because they are both white, and they are different because one is bigger than the other.)

Pair each child with a partner and have pairs spend a few minutes comparing their buttons. Then have the children take turns sharing their comparisons. On chart paper, keep track of the comparisons.

When the students have all shared their comparisons using the language of *same* and *different*, tell them you're going to use their ideas to think about ways to sort buttons.

Ask the children to hold their buttons in the palm of their hands. Explain that you are going to repeat some of the ideas that people have shared. If the sorting idea is true for a particular child's button, that child should place his or her button on the floor in front of him or her.

Call out some of the sorting criteria that the students cited when working with partners—for example, round buttons, buttons with two holes, white buttons, and so on. As the children begin laying down their buttons, start using comparison words such as *most, more,* and *fewer.* For example, "Most of the buttons are round" or "There are fewer buttons with four holes than two holes." After each attribute is called out, the children pick up their buttons and decide again if the next attribute holds true for their particular button.

Explain to the children that the button set will be available every day during math time and they are free to explore it. Remind the children that they are to sort the buttons by what they have in common, such as color and shape, and that these sorting ideas will work for a lot of other objects as well.

Replace the buttons in the bag while discussing with the children the importance of checking to ensure the bag is secure. Show the children where this and other sorting sets will be kept.

Grade K Overview

Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

- Work with numbers 11–19 to gain foundations for place value.

Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in categories.

Geometry

- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade K

Counting and Cardinality K.CC

Know number names and the count sequence.

1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
    a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
    b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
    c. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Compare numbers.

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]
7. Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5. Fluently add and subtract within 5.

[1]Include groups with up to ten objects.
[2]Drawings need not show details, but should show the mathematics in the problem.
(This applies wherever drawings are mentioned in the Standards.)

Number and Operations in Base Ten K.NBT

Work with numbers 11–19 to gain foundations for place value.

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data K.MD

Describe and compare measurable attributes.

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Classify objects and count the number of objects in each category.

3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[3]
4. ***Demonstrate an understanding of concepts time (e.g., morning, afternoon, evening, today, yesterday, tomorrow, week, year) and tools that measure time (e.g., clock, calendar). (CA-Standard MG 1.2)***
   a. ***Name the days of the week. (CA-Standard MG 1.3)***
   b. ***Identify the time (to the nearest hour) of everyday events (e.g., lunch time is 12 o'clock, bedtime is 8 o'clock at night). (CA-Standard MG 1.4)***

Geometry K.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.
2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

[3]Limit category counts to be less than or equal to 10.

Grade 1 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Understand and apply properties of operations and the relationship between addition and subtraction.
- Add and subtract within 20.
- Work with addition and subtraction equations.

Number and Operations in Base Ten

- Extend the counting sequence.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure lengths indirectly and by iterating length units.
- Tell and write time.
- Represent and interpret data.

Geometry

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 1

Operations and Algebraic Thinking 1.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]
2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.

3. Apply properties of operations as strategies to add and subtract.[3] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*
4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

Add and subtract within 20.

5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with addition and subtraction equations.

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

7.1 Write and solve number sentences from problem situations that express relationships involving addition and subtraction within 20.

8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = – 3, 6 + 6 = .*

Number and Operations in Base Ten 1.NBT

Extend the counting sequence.

1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:
   a. 10 can be thought of as a bundle of ten ones — called a “ten.”
   b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
   c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

[2]See Glossary, Table 1.
[3]Students need not use formal terms for these properties.

Use place value understanding and properties of operations to add and subtract.

4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Measurement and Data 1.MD

Measure lengths indirectly and by iterating length units.

1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.
2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

Tell and write time.

3. Tell and write time in hours and half-hours using analog and digital clocks.

3.1 Relate time to events (e.g., before/after, shorter/longer).

Represent and interpret data.

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

4.1 Describe, extend, and explain ways to get to a next element in simple repeating patterns (e.g., rhythmic, numeric, color, and shape). (CA-Standard SDAP 2.1)

Geometry 1.G

Reason with shapes and their attributes.

1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[4]
3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

[4]Students do not need to learn formal names such as “right rectangular prism.”

Grade 2 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and interpret data.

Geometry

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 2

Operations and Algebraic Thinking 2.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
5. ***Use repeated addition and counting by multiples to demonstrate multiplication.***
6. ***Use repeated subtraction and equal group sharing to demonstrate division.***

Number and Operations in Base Ten 2.NBT

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:
   a. 100 can be thought of as a bundle of ten tens — called a "hundred."
   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
2. Count within 1000; skip-count by **2s**, 5s, 10s, and 100s.
3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
6. Add up to four two-digit numbers using strategies based on place value and properties of operations.
7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

7.1 Use estimation strategies in computation and problem solving with numbers up to 1000.

7.2 Make reasonable estimates when adding or subtracting.

8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
9. Explain why addition and subtraction strategies work, using place value and the properties of operations.[3]

[1]See Glossary, Table 1.
[2]See standard 1.OA.6 for a list of mental strategies.
[3]Explanations may be supported by drawings or objects.

Measurement and Data 2.MD

Measure and estimate lengths in standard units.

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
3. Estimate lengths using units of inches, feet, centimeters, and meters.

3.1 Verify reasonableness of the estimate when working with measurements (e.g., closest inch). (CA-Standard NS 6.1)

4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. ***Know relationships of time (e.g., minutes in an hour, days in a month, weeks in a year).***
8. Solve word problems involving ***combinations of*** dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Represent and interpret data.

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[4] using information presented in a bar graph.

Geometry 2.G

Reason with shapes and their attributes.

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[5] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves*, *thirds*, *half of*, *a third of*, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

[4]See Glossary, Table 1.
[5]Sizes are compared directly or visually, not compared by measuring.

Chasing Vermeer

A Lesson for Third and Fourth Graders

by Maryann Wicket

From Online Newsletter Issue Number 21, Spring 2006

In Blue Balliett's novel Chasing Vermeer *(Scholastic, 2005), Petra and Calder, the main characters, are in the same class but barely know each other. Their friendship grows, however, and they work together to recover a stolen painting—a valuable Vermeer. Pentominoes are included in the clues they need to decode. Maryann Wickett uses this book as a springboard for a lesson in which students visualize geometric shapes, create all possible five-piece pentomino arrangements, and then go on to two other activities involving pentominoes.*

I began by reading aloud to my class of third and fourth graders the two introductory pages from *Chasing Vermeer*—"About Pentominoes and About the Story" and "About the Artwork: A Challenge to the Reader"—and then asked students for their reactions.

"Sounds like a mystery to me," Traci said. "It's mysterious when the author says, 'Pentominoes, like people, can surprise you.' I don't even know what pentominoes are."

"I bet we'll find out! Otherwise there wouldn't be a special page about them," commented Sara.

"Pentominoes can make a code! That's cool," Greg said.

When all who wanted to had shared their reactions, I read aloud the novel's first two chapters. At times the students insisted on sharing their reactions, especially as three letters asking for the receiver's help were mysteriously delivered to three as yet unknown recipients.

We carefully examined the illustration on page 5, looking for a certain living creature that might help us discover the hidden message described in the introductory pages. I had two copies of the book so the students were able to gather round and see more easily. After a few moments, Carlos said, "Hey, look, there are two frogs! Maybe that's the creature."

Vanessa carefully examined the bottom illustration on page 5, pointed to the envelope in the woman's hand, and said, "That L on the envelope could be a pentomino piece." We quickly checked the introductory pages and verified that the L-shaped figure Vanessa noticed could be the V pentomino piece.

I said, "Hmm, two frogs and the V. Very interesting. I wonder what it all means. Are they clues?"

Next, I provided students with 1-inch tiles and 1-inch graph paper so each could make his or her own set of pentominoes. I explained that pentomino pieces were made of five tiles and whenever two sides touched, they had to match exactly. I drew on the board two samples—one that was considered acceptable and another that wasn't.

This activity had two purposes: first, to provide students with experiences visualizing geometric arrangements and, second, to have students figure out when all twelve possible arrangements had been found, which requires logical thinking. As students worked, I circulated, observing their work and asking questions.

When students felt they had all possible arrangements, I asked them to verify this. Trevor and Betsie used their tiles to demonstrate how they first built a straight line of five tiles. Then they moved the top tile around the other four tiles, recording all the possible arrangements.

Bonnie watched as they explained what they had done. She noticed, however, that some arrangements were the same if they were flipped or rotated. Bonnie's observation took Trevor and Betsie by surprise, and they began to explore further which of their pentomino arrangements, if flipped or rotated, were duplicated. It took the remainder of the period for the children to complete their sets of pentominoes.

Day 2

I reread the second paragraph of page 13 of *Chasing Vermeer.* As Calder had done in this paragraph, I challenged the students to use their pentominoes and work with a partner to find possible six-piece pentomino rectangles. When students found a way to make a six-piece rectangle, I gave them the choice of finding another way or trying to discover how to use all twelve of their pentominoes to make a rectangle.

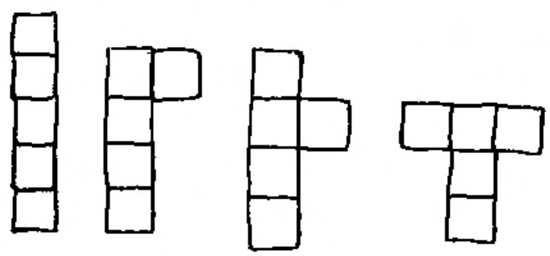
I left enough time to read aloud Chapters 3 and 4 and to carefully examine the illustrations for other hidden frogs and pentomino pieces. The students found a frog in the illustration on page 29 but no pentomino. Chapter 4 showed Calder with his pentominoes spread out on his desk but no frogs. When Calder pulled the P pentomino from his pocket on page 24, he noticed Petra walking in front of him. Anya commented, "Hey, P is for *Petra* and for *pentomino*."

Mark added, "I think Calder's last name starts with P too." We looked back to the previous page, page 23, and confirmed that Calder's last name, Pillay, did start with P.

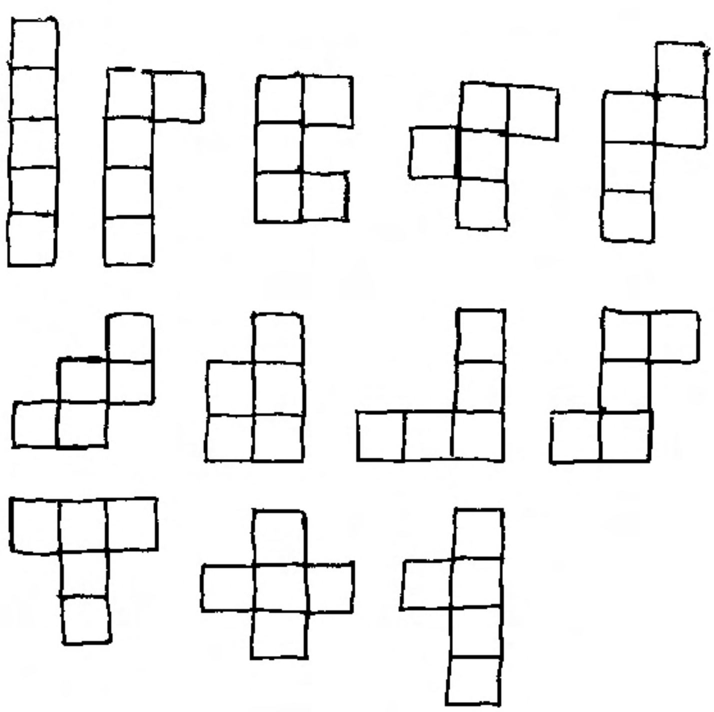
"Hey, that's weird," Miguel said. "It's like all these pieces of the story are being linked by pentominoes, just like it said at the very beginning." (Miguel was referring to the following quote from the introductory page, "About Pentominoes and About This Story": "This book begins, like a set of pentominoes, with separate pieces. Eventually they will all come together.") Several others nodded in agreement.

Day 3

I started class by reading Chapter 5. The students noticed immediately that the fur collar illustrated on page 48 had both a pentomino and a frog. To give students more experience with the pentomino pieces so it would be easier to spot them in the illustrations, I asked the children to get their pentominoes and sit on the floor so they could see the pentomino pieces and the tiles I had put in front of me on the floor. I gave them the following challenge: “Arrange your pentomino pieces into a line, so that only one square needs to be moved to change a shape into the one next to it.” I explained that the tiles could be useful for helping them think about the challenge. I used the tiles to quickly demonstrate the instructions.



The students worked in pairs and came up with several solutions. As they shared their solutions, I recorded them on a class chart.



I ended the class by reading Chapter 6. This was very exciting because on page 57, there is a code involving pentominoes and some messages to be decoded. The students were eager to decode the messages. I told them I would post the code and the messages for the next day.

We continued to return to the book often over the next several weeks. We found other messages to decode, made up our own codes, examined the illustrations, searching for frogs and pentominoes, noticed many odd coincidences, looked for pentomino shapes in the world around us, discovered that the number twelve was important to the story, and tried to figure out the message in the illustrations. There are additional ideas for math activities in the back of some of the editions of the book as well as online at *www.scholastic.com/chasingvermeer.*

Appointment Clock Buddies

Name ______________________

Date ______________________

12:00 Buddy

____

9:00 Buddy

____

3:00 Buddy

____

6:00 Buddy

____

Appointment Clock Buddies

Teacher Directions

Overview: This activity is a fun way to create ready-made sets of partners for cooperative learning lessons.

Steps:

1. Duplicate one copy of the Appointment Clock printable for each student and distribute prior to the activity.
2. To begin, ask students to write their own names and the date at the top of their clock papers.
3. Then ask students to stand up with their papers and a pencil and begin moving quietly around the room.
4. Give an audible signal (clap, chimes, buzzer, bell, etc.) and have students stop and find a partner.
5. When everyone has a partner, ask them to sign their partner's paper on the 3 o'clock line. It's very important that they all start on the same line.
6. Give the signal to move and have students mix around the room. Signal them to stop and find a new partner. This time you'll need to check to be sure everyone does have a new partner before you proceed with the next step. You may have to switch a few students to create new pairs.
7. Now ask students to sign each other's papers on the 6 o'clock line.
8. To complete the activity, have students mix around and pair up two more times. They'll sign next on the 9 o'clock line and finally on the 12 o'clock line. If you have a small class, it may take some help on your part to make sure everyone has a new partner for each time slot.
9. After all lines are signed, students return to their seats and put their papers in a safe place such as in the front of their binders.
10. Now each time you need for them to form partners, have them take out their appointment clocks and name one of the four times. For example, "Today you will meet with your 9 o'clock appointment to complete your graphic organizer."

Note: Each time you use the clock, remind students to put their clocks away in a safe place. After half a dozen kids have lost their clocks, this method of pairing becomes ineffective! Then it's time to start over and make new clocks.

BUILDING COMMUNITY IN THE CLASSROOM

by Mishael Hittie

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As teachers, we have a very difficult job. We are to take a group of diverse students and teach them in ways that meet all of their needs on every level: academic, emotional, and social. Yet, students come to us with many different strengths and abilities. They also come to us with pain and hurt in their lives. Our challenge is to create a classroom culture that builds on their strengths and heals their hurts so that learning can occur.

By definition, a community is a group of people who work with one another building a sense of trust, care, and support. This means that in our classrooms, part of our job is to provide opportunities and structures by which students can help and support one another. It also means that we provide explicit instruction and support so that students learn *how* to do this. As I've worked to build community in my classroom, I've realized that it is connected with everything I do. However I have found a way to structure how I think about building community.

Helping students building 'community'.

- Fostering community awareness by creating structures that emphasize collaborative activity and joint problem-solving.
- Modeling the language of cooperation (eg. 'friends help friends. We all work together'.)
- Extending community-building efforts to all areas of classroom activity, finding places where students can assist each other.
- Reinforcing the notion that all ideas in the community are respected and valued.
- Providing visible cues throughout the classroom that encourage collaboration and cooperation (eg. Posters highlighting a 'Cooperative Work Ethic; seating arrangements).
- Strategic planning for the arrival of new students to the classroom (eg. Assigning a friend to each new student to provide support, guidance, and leadership).
- Modeling collaboration, making teachers' collaborative planning activities visible to students.
- Play group games that foster community building and read literature that teaches these topics.

Provide Voice

Making choices. Children both need help from adults to provide structure in their lives and freedom to make choices. Students need to take responsibility for choices as part of their growing and learning. I establish many ways to give students reasonable choices on a daily basis. Often I present options to the entire group and allow students to select through a democratic show of hands. Some other examples include:

What to learn. At the beginning of the year and at the beginning of each unit, I find out what students would like to learn and incorporate their ideas into my plans.

What order: Students sometimes choose order of work activities. Should they learn the new lesson or go over homework first? Do reading workshop or writing workshop first?

Choice Time. I structure time every day for students to participate in any of several specified choice activities. This is not "free time," as the activities are all structured around a topic, such as literacy, science, or social studies.

What to read/write. Students are given choices about what books to read or what stories to write. Even when I select the genre to read or write, the student selects the content.

Day to day goals. Kids choose weekly goals to set for themselves in terms of either academics or behaviors. They are required to think reflectively to choose goals that they think need improvement. Again, this is their choice not me demanding.

Where to sit. For most activities, where to sit is not really the key issue. I let students choose where to sit, as long as it does not impede their work process.

It is also very important for children to feel like the class is theirs and not just the teachers. This adds to a feeling of responsibility and helps children learn to take care of their things. As I get started at the beginning of the year, I remember that first impressions, for adults as well as kids, can be lasting ones. I seek from the beginning to set the tone and build a culture of mutual support in our class. Some ways I encourage this feeling early on are to decide as a group how the room will be set up (this year I am holding a room decorating party), what the bulletin boards should be for, and what the classroom rules should be. With little direction, students often come up with the same ideas I would have chosen. Sometimes they surprise me and come up with better ones.

Be Friends

It is very important for children to get to know each other and see each other as individual people. They can learn to respect and appreciate differences and enjoy things that are the same. This adds a great deal to the feeling that we are a family of people here to learn together. Some things to do to encourage this are:

- Instead of taking general recess time, set aside some time during the day to play a game together.
- On Mondays share their weekends and other things that make them happy or worry them.
- At the beginning of each day, hour, right after recess, or wherever it seems to work best have children talk with a friends about anything they want to. This gives them a chance to share feelings with people.
- Do fun work together, like art projects
- Arrange activites with their families outside of the normal school day.
- Play educational games together.

Many teachers, including myself, take the first two weeks of school to really concentrate on getting to know each other and building community. This foundation serves well to prevent future problems and provide a respectful way of dealing with those problems that do occur.

Promote Respect

Daily routines. In my experience, we either make or break community in the smallest of daily routines. We either treat simple issues in ways that build respect and teach responsibility or in ways that are controlling. In elementary schools, going to the bathroom is an excellent example. I see one class lined up at the bathroom. The teacher is admonishing the children in line to stay quiet, which only a few of them were doing, and is trying to rush the children through the bathroom. The noise of laughter and scuffling around is easily heard through the door. In about 20 minutes, the teacher manages to get all of the children out of the bathroom and back to the class to work. Sound familiar?

In my class, the children wave one of two bathroom passes to ask permission to leave without interrupting and I nod yes. I check the clock to make sure they are not gone too long. Most of the children go to the bathroom without outsiders really knowing that they were gone. Many children even try to wait longer so they won't miss fun things in class.This solution teaches children to be responsible for their own actions, while not wasting learning time.

These and other issues can have a significant impact on the community feeling in the classroom. If we are teaching students how to function as responsible adults, not controlled children or inhabitants of institutions, we must develop strategies by which students are given daily responsibility and supportive structures for them to do so. In each basic daily routine I think about ways to make it fun and teach responsibility at the same time. Here are typical routines and some strategies that build community and responsibility.

Getting class attention. Clap hands in a rhythm students imitate, ring a bell or other musical instrument, hold up a silent hand and count down while children join in, or quietly ask each group to put their eyes on the teacher. Involve the kids themselves in figuring out attention getting strategies.

Transitions between areas or activities. Begin singing a song all know, ask kids to fill in blanks in funny story, start reading a poem, begin a riddle.

Bathroom. Have a signal to ask permission silently, use a bathroom pass, provide consequences for misuse.

Lining Up. Have children lead with teacher following, stopping at key areas.

Lunch Count. Use a magnetic board with one column for hot and cold lunches. Have students move their magnetic ticket to hot/cold lunch and another student charged with the job of pulling tickets and sending information to the office.

Dismissal. Set a routine for leaving room. Assign jobs needed for straightening up.

Attendance. Students take attendance on a sheet, passing it around,or student checks off names on a printed list.

Tone of Voice: "When you are done writing please match eyes with me and I know we can go on."This is an example of a respectful strategy by which to interact with students in the class moving from topic to topic. Kids need to feel accepted in class and it is very important not to embarrass students or put them on the spot. Anytime students don't want to talk, they just say 'pass'". Then they will be more likely to respond at a later time. If they pass all the time, I talk with them individually. I try to always talk in a respectful manner, so that I am modeling what

I want them to do. If I yell at them, doesn't it make sense that they will be rude back?

Difficult Students: We probably send the biggest messages about respect to all of our students when they see us handle the most difficult students. In my fourth grade class two years ago, I was having difficulty with a girl who was arguing insistently. While I tried to stay calm, I finally began yelling. Another student mumbled, "Great! Now she will be mad at all of us. There goes this day." When other students see that I treat all students with respect, they are more comfortable taking risks and sharing things about themselves. However, when they feel intimidated because they are worried about what I might do, then the strength of the community bond is broken. I strive to be calm and respectful and rely on the classroom structures put into place to help things run smoothly.

Another Strategy: When you know you cannot keep calm with a student anymore, give yourself a break. Arrange for them to visit another classroom, run errands for the office, or sit in a cooling off spot in the room. Do whatever works for you and that child to give each other some space. Then when everyone is calmer, come back and talk to the child.

Work Together and Problem Solve

Class Belongs to Everyone. Students need to understand that the classroom belongs to everyone, not just the teacher. This does not happen by accident but takes careful thought and consideration. In addition to the basics of arranging and decorating the room, students need opportunities to help create the rules that will govern their lives in the classroom. Creating classroom rules together, where everyone's ideas are listed, then grouped into three or four rules that are easy to remember, is essential to establishing a feeling of ownership. This requires that teachers be willing to give up the feeling of total control and replace it with a feeling of pride in teaching kids how to control themselves. Once these rules are agreed upon, they are posted and referenced often. I might say, "Johnny that breaks the agreement you made to abide by rule #4". This puts the responsibility for actions on the student, and reminds other students of our agreements together.

Class meetings. Classroom meetings can be valuable tools for democratically handling issues that arise. In my class, we have meetings almost every day, whether it is about teasing, staying on task in writing workshop, or turning in homework. Meetings vary in lengths, sometimes taking only ten minutes and sometimes longer, depending upon the issue.

The topics of classroom meetings are chosen both by students and myself. I make available a box in which the students or teacher can anonymously place issues. This allows even those who are not comfortable sharing aloud to have their needs addressed.

Classroom meetings are run by the children, a shift which is very difficult for some teachers. In the classroom meeting, students take turns in the different roles of notetaker and moderator. I establish basic rules that allow students to speak one at a time. We pass around a designated object (a stuffed penguin in my room) to the person who has the floor to speak. Students are not required to speak and may pass if they so choose. Most importantly, the class makes a decision that results in action. My students very quickly loose interest if they do not feel that their decisions are taken seriously.

Cooperative learning. Working together raises the level of capability from what children can do on their own. Create structures that enable children to work together in any subject, whether through reading buddies, editing partners, or clock partners in math. Create projects that require students to work together, but have jobs at different levels of expertise. This enables every student to have a job that others depend upon but at their own level. In these structures, they learn that they can depend on others for help and they learn that different people bring different strengths to a partnership

Clock Partners. A strategy that works from kindergarten through high school is to use clock partners to pair students for activities. A large clock has a line in the middle of the face, and lines at each hour. Students ask each other to sign on each hour line. They cannot have the same student twice. My rule is that if someone asks, the student cannot say no. In an activity, I ask students to get their clocks and get with their 5:00 partner to do work together. This simple strategy removes aggravations often associated with pairing students. No more hurt feelings, complaints about not getting their way, or trying to separate the same students who always work together.

Sharing work. Sharing completed work gives students a sense of appreciation for others and a sense of pride in their own accomplishments. It builds self-esteem and community all at the same time. When a genre of writing has been completed, I have had a book sharing time or shared stories with other classes, either older or younger. This year I am going to try having a celebration at the end of a unit, where children prepare activities, set up projects, skits, and written work to show what they have learned. We will invite other classes or parents to participate.

Circle of friends. Sometimes I need intentional strategies to support students. Circle of friends is a powerful strategy by which I do this. In a circle of friends, one student is seeking help and support from others. While the idea of circles was born as a way to help students with disabilities, it is a powerful tool for anyone.

In situations where I see students having a real tough time, I ask them if they would like to have a circle of support. Students decide who to invite to be part of their circle. Sometimes they have specific people. A student has also, with my help, opened the invitation to all students within a class. Students need the assistance of an adult who acts as a facilitator for the group. I meet with my children at lunch time once a week, but it could be handled in other manners. This year, I am going to train a child in my existing group to facilitate the meeting,so that they can also conduct circles during the school day.

Once the group is selected and a meeting time and place established, the group meets. I start with the MAPS process to focus the work of the circle group. We explore the dreams and fears of the student and develop an action plan, including support and assistance from the circle group, to help the person cope with challenges and move ahead to desired outcomes.

Kids have a natural desire to help, and I channel that to more specifically meet my children's needs. Given proper modeling and support, children can meet very high expectations.

Student Leadership

In my classroom, I want students to become leaders and extend themselves outside of their comfort zone to help others reach their full potential. As I find ways for students to exercise leadership roles I increase their understanding of the subject of study, build responsibility, and raise their self-esteem.

Jobs: In my class, every kid has a job, one for which they are responsible without constant reminders..This helps students see the interdependence people have on each other.

Other Roles. Students can take other leadership roles. As I give students choices, leadership roles often evolve naturally out of their work. I have learned to rely on some students as peer mediators, as members of circles of support, as particularly good at giving comfort, o rat leading classroom discussions. Students help me design fun lessons and give me ideas for students having difficulty. The more we can involve our students in real decisions, the more they learn and the more help we all have.

Experts. Students need experience teaching others what they know. As teachers, we know that this is not a simple task. To teach someone else, children must have a much deeper understanding of their subject matter. There are many adults who in their own professions cannot articulate what they are doing and why. Having children explain their thought process to others will teach a much needed skill, and will create a feeling that everyone is an expert. For example, a student was writing a story but was not using needed quotation marks. I helped her look for examples in books. When she shared what she had done, I asked her if she would be willing to help others. She said that she would. Later another student asked me for help with quotation marks. I sent that student to my expert. For the rest of the year, they were the teachers for quotation marks.

I also facilitate this process by creating a “yellow pages”. Students identify 2-3 skills in which they excell and write an advertisement for themselves. These are compiled into a class yellow pages. Students often consult this book when they needed help on topics. This gives every child the opportunity to be the teacher, including students who most often were seen as needing help from others

Students Lead Lessons. My students love to play teacher. When they have seen a teaching strategy several times, I will let them take turns coming to the board and being the teacher. I sit in adesk just like a student and raise my hnad with quesitons. Not only does this give the students an excellent learning opportunity that is fun, it is an excellent way to get a window into their thinking about the subject.

Raise Self Esteem. I cultivate leadership by asking students to help others after their work is finished or teaching a few students a new skill that the whole class needs to learn and then send others to them for help. This is an excellent way for those children who normally finish last to have some experience being the leader. I provide guidelines for how helping looks different from doing the work for the other person. Once this is accomplished, the room is transformed into a place where there are many people to ask for help, not one teacher who cannot be everywhere at once.

The range of what children can teach each other is amazing. Experts can be on any subject from spelling, adding or dividing, proving theorems, placing capital letters or quotation marks, making a cursive ‘m’, to being knowledgeable about the science topic being studied. The key is to teach the kids share their knowledge with others and accept knowledge from others as well.

Individualize and Discuss It

Ability differences up front. It is important to help students understand that we all have different abilities and that this difference does not make us better or worse people. It is important for every student to get what they need to learn, but this does not mean that they all need or get the same thing. I start talking about this with my students by teaching them about multiple intelligences This helps students who know they function lower in some areas to look at their strengths in other areas. I talk about how I teach in my class to help them understand that different students will be working at different levels. By watching my interactions with classmates who are having difficulty, students learn not to ostracize others for their abilities. It becomes ok to be different, and students learn to look for one another's abilities in different areas. They learn how to work in groups with people who have very different abilities, and this skill will serve them well later in life.

At first the idea of building community in our class and in our school may seem just a bit like a fairy tale. After all, many of us experienced schools where feelings were not considered important, we sat in rigid rows and listened to boring lectures about topics we were told were good for us. Successfully build a collaborative community of 3rd graders, 8th graders, or high school seniors may seem a bit far fetched. Yet, teachers all over the world are building a literature of practices that far extend the beginnings I have sketched and it is growing by leaps and bounds. With the world growing more violent every day, there is a real need for feeling part of a community and knowing how to create it. So it becomes part of our journey as teachers that's both full of excitement and many frustrations, but we'll find not only is it our key to successful teaching, it contributes a richness to our lives that would be impossible without our students help.

Multiple Intelligences

From Peterson and Hittie (In Press.) Teaching in the Inclusive School
Allyn & Bacon.

Multiple Intelligence Description	Think. . . Love . . . Needs . . .	Teaching Menu (a few ideas)
1. Linguistic: the capacity to use language to express ourselves and to understand other people. Examples: poet, writer, orator, lawyer, teacher.	In words . . . reading, writing, telling stories, playing word games . . . Books, tapes, writing tools, paper, diaries, dialogue.	Use storytelling to explain . . . Conduct a debate on . . . Write a poem, legend, short play or news article about . . . Conduct an interview about . . .
2. Logical-mathematical: ability to use numbers effectively and to reason well logically. Examples: mathematician, accountant, computer programmer, scientist.	By reasoning . . . experimenting, questioning, figuring out logical puzzles . . . things to explore and think about, science materials, manipulatives.	Translate a . . . into a math formula. Design & conduct an experiment on . . . Make up syllogisms to explain . Describe patterns of symmetry in . . .
3. Spatial: competence to represent the spatial world internally in your mind and to use materials to impact the environment. Examples: hunter, scout, artist, architect, inventor.	In images and pictures. . . designing, drawing, visualizing, doodling . . . art, video, movies, imagination games, mazes, illustrated books, trips to art museums.	Chart, map, or graph . . . Create a slide show, video, or photo album of . . . Create a piece of art that illustrates . . . Draw, paint, sketch or sculpt . .
4. Bodily-kinesthetic: expertise in using one's whole body to express ideas and feelings and capability to use one's body to make or change things. Examples: actor, athlete, sculptor, mechanic, surgeon.	Through bodily sensations. . . dancing, running, jumping, building, touching. . . role play, drama, movement, constructing, sports, hands-on learning.	Create a sequence of movements to explain. . . Build or construct a . . . Plan and attend a field trip to. . . Bring hands-on materials to demonstrate. . .
5. Musical: proficiency to think in music, hear patterns, recognize them, remember them, manipulate them. Examples: singer, song-writer, music critic.	Via rhythms and melodies . . . singing, whistling, humming, tapping feet. . . sing-along time, music playing, musical instruments, music.	Give presentation on . . . with musical accompaniment. Sing a rap or song that explains Explain how the music of a song is similar to . . . Make an instrument and use it to demonstrate. . .
6. Interpersonal: ability to understand thoughts, feelings, motivations of other people and to interact well with them. Examples: politician, salesperson	By talking with other people. . . leading, organizing, talking, mediating, partying. . . friends, group games, social events, mentors	Conduct a meeting to address. . Participate in a service project to Teach someone about . . . Practice giving and receiving feedback on . . .
7. Intra-personal: understanding oneself – feelings, reactions to others – and acting on that understanding. Aware of inner moods, capable of self-discipline, and deeply reflective. Example: philosophers, poets, counselors.	By reflecting deeply inside themselves. . . setting goals, meditating, dreaming, being quiet. . . secret places, time alone, self-paced projects, choices.	Describe qualities you have that will help you . . . Develop a plan to . . . Describe a personal value about . Write a journal entry on . . . Assess your own work in . . .
8. Naturalist: highly sensitive and responsive to living beings (plants, animals), the natural world, and the environment. Examples: "street smart" students, hunters, farmers, botanist.	By interacting with nature and the environment . . . camping, moving around the community, organizing the environment . . . time in nature or the community, organizing events.	Create observation notebooks of Describe changes in the local community. . . Care for pets, wildlife, gardens, or parks in . . . Draw or photograph natural objects or the community.

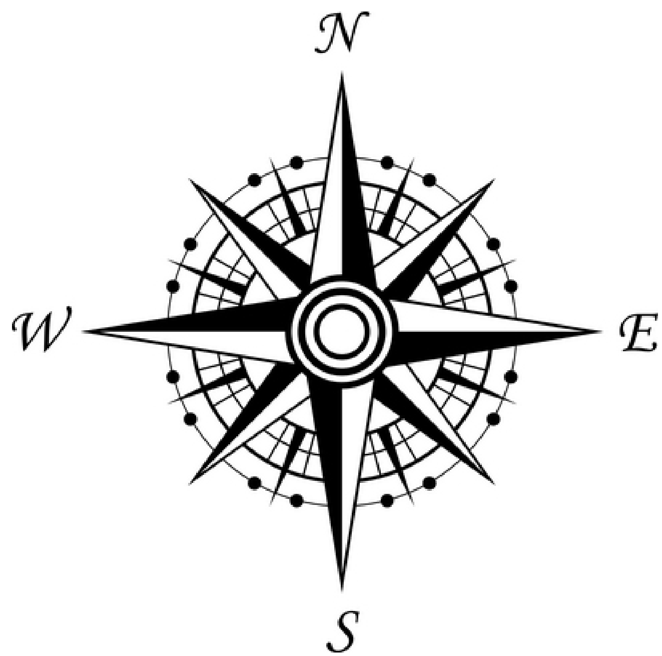
Compass Buddies

Name ______________________________

Date ______________________________

North Buddy

West Buddy



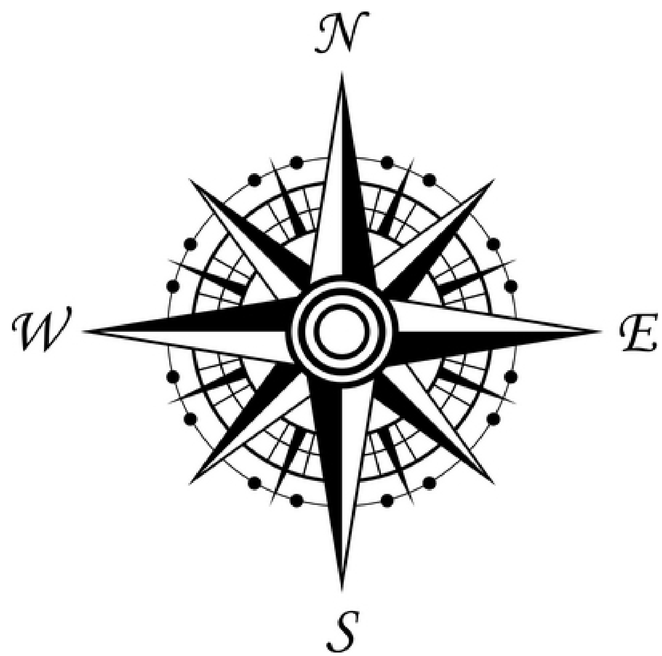
East Buddy

South Buddy

Compass Buddies

Name ______________________________

Date ______________________________



Compass Buddies

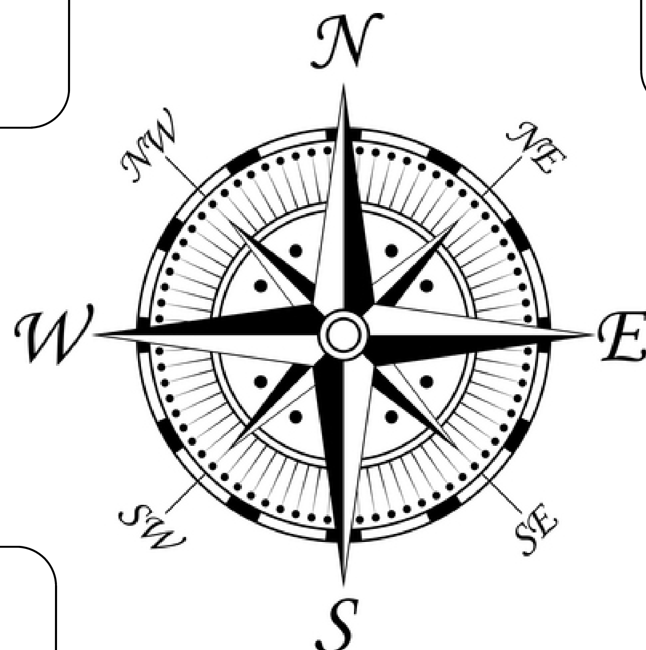
Name ______________________

Date ______________________

North Buddy

Northwest Buddy

Northeast Buddy



West Buddy

East Buddy

Southwest Buddy

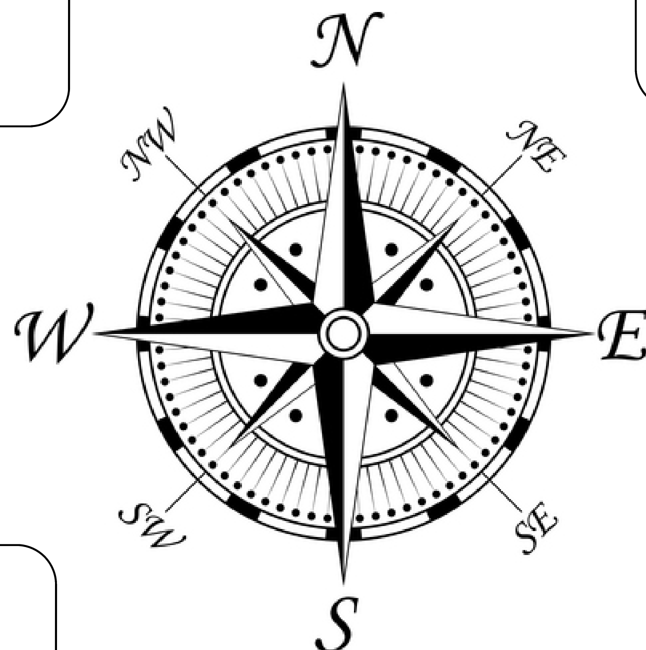
Southeast Buddy

South Buddy

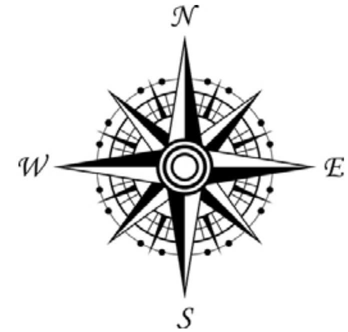
Compass Buddies

Name ______________________________

Date ______________________________



Compass Buddies



Teacher Directions

Overview: This is a fun way to create ready-made sets of partners for cooperative learning lessons.

Steps:

1. Decide which one of the Compass Buddies printables you plan to use, and duplicate one copy for each student. The compass rose with the four cardinal directions is easier to use, and I suggest using it the first time you do the activity. (Note: If you duplicate the form on colored paper, it's easier for kids to find it later.)
2. To begin, distribute one copy to each student and ask them to write their own names and the date at the top of the page.
3. Next, ask students to stand up with their papers and a pencil and begin moving quietly around the room.
4. Give an audible signal (clap, chimes, buzzer, bell, etc.) and have students stop and find a partner.
5. When everyone has a partner, ask them to sign their partner's paper on the **North Buddy line**. It's very important that they all start on the same line or the follow-up activity will not work.
6. Give the signal to move and have students mix around the room. Signal them to stop and find a new partner. This time you'll need to check to be sure everyone does have a new partner before you proceed with the next step. You may have to switch a few students to create new pairs.
7. Now ask students to sign each other's papers on the **East Buddy line.**
8. To complete the activity, have students mix around and pair up two more times. They'll sign next on the **South Buddy line** and finally on the **West Buddy line.** If you have a small class, it may take some help on your part to make sure everyone has a new partner for each time slot. If you are using the printable that has all eight directions, it will be nearly impossible for students to have brand new partners each time, so just tell them that no name can appear more than twice.
9. After they finish, students return to their seats and put their papers in a safe place such as in the front of their binders. You might consider collecting the papers and storing them for future use.
10. Now each time you need for them to form partners, have them take out their Compass Buddies form and name one of the four directions. For example, "Today you will meet with your West Buddy to complete your graphic organizer."

Note: Each time you use the Compass Buddies form, remind students to put their clocks away in a safe place. After half a dozen kids have lost their forms, this method of pairing becomes ineffective!

Thanks to Monica Horn for suggesting the Compass Buddies variation of the well-known Clock Buddies activity!

Math Solutions
PROFESSIONAL DEVELOPMENT

Counting Crocodiles

A Lesson with Kindergartners

by Andrea Holmes

featured in *Math Solutions Online Newsletter*, Summer 2008, Issue 30

Andrea Holmes knew that the kindergarten children in Mrs. Fisher's class had had numerous opportunities to listen to and engage with counting books. She also knew that additional practice allows children to continually think about numbers from one to ten and provides a glimpse of the problem-solving strategies children rely upon most. To further the children's experience with number and provide an assessment opportunity for herself, Andrea chose to use this lesson based on the children's book Counting Crocodiles, by Judy Sierra (Harcourt Brace, 1997).

In this rhyming tale, a hungry but clever little monkey is stranded on her island in the Sillabobble Sea with nothing but lemons to eat. She spies a banana tree on another island across the sea, but, to her dismay, the sea is filled with vicious crocodiles. Climbing up the lemon tree, she slyly says, "I wonder, are there more crocodiles in the sea, or monkeys on the shore?"

An engaged crocodile approaches, saying, "Just look at us! I have a hunch you've never seen a bigger bunch," challenging Monkey to count them. Not quite as clever, Crocodile intends to eat Monkey for lunch, but only after he is able to prove to her that there are indeed more crocodiles than monkeys. As the crocodiles line up for Monkey to count, they unknowingly create a bridge for her to cross, enabling Monkey to reach the island with the banana tree. Ready to eat her now that she has reached the other island, the crocodiles ask, "How many of us did you find?" With her mouth full of bananas, she convinces the crocodiles that she needs to count them one more time to be sure, allowing her to make it back to her island. This lesson engages students in counting and adding the numbers one through ten.

Before reading *Counting Crocodiles*, I explained to the children that this story was about a clever little monkey who plays a trick on some crocodiles. I asked them to think about what she does that is so clever and how she goes about tricking the crocodiles. To set the stage for the problem-solving situation that would follow the story, I asked them to think about how many crocodiles the monkey tricks. The children were immediately engaged in the story, drawn to its rhyming nature. Throughout the reading, I checked on their thinking by asking them to show thumbs up or down if they had an idea about the trick Monkey played and about how many crocodiles there were so far.

After the story, I asked the children several questions to get them thinking about the story. "Why do you think Monkey wanted to cross the Sillabobble Sea to get to the other island?" To avoid having children shout out their responses, I reminded them to first answer my questions in their heads.

After a moment, I asked them to share with a neighbor sitting next to them. Sharing with a neighbor ensures active participation of all the children. To begin the discussion, I asked Erin to share why Monkey wanted to cross the Sillabobble Sea. "She wants the bananas!" Erin explained. The rest of the children responded with a thumbs-up sign to support Erin's answer.

I continued. "Why did Monkey ask, 'I wonder, are there more crocodiles in the sea, or monkeys on the shore?'"

Following the same process, I called on Max, who replied, "She wanted the crocodiles to help her get across the sea." Again, a majority of the students gave a thumbs-up.

"What is Monkey cleverly doing as she counts the crocodiles?" I inquired.

"Tricking them by getting them to make a bridge," Tate eagerly answered.

"After Monkey has counted the crocodiles, why do you think she says that she needs to count them one more time?"

"She needs to get back to her lemon tree," Karissa told the class.

The children had demonstrated their understanding of the story by thinking, sharing with their neighbors, and, finally, explaining their thoughts to the class. Next I encouraged them to think about the question I had posed before reading the story. "How do you think we can figure out how many crocodiles were in the sea altogether?"

Using strategies from *Comprehending Math: Adapting Reading Strategies to Teach Mathematics, K–6,* by Arthur Hyde (Heinemann, 2006), I asked, "What do we know for sure about this problem?"

"We know there were a lot of crocodiles," Sophia offered.

"That's right," I responded. "Are there any clues that tell us about the number of crocodiles?"

Chase eagerly shared, "There are three hundred!" His answer helped me realize I needed to provide more scaffolding.

I showed the students the page where Monkey starts counting the crocodiles. "How many crocs does Monkey count on this page?" I asked.

In unison this time, the class said, "One crocodile!"

At the top of the first column in a three-column chart, I wrote the heading *What We Know About This Problem.* Under that heading, I wrote *one crocodile.* I turned the page and the children said, "Now two crocodiles." I added that to the chart. The children continued to tell me how many crocodiles Monkey counted on each page. After I wrote *ten crocodiles,* Bryce pointed out that there were a lot of crocodiles to count.

"What are we trying to find out with this problem?" I asked.

The children paused to think before they raised their hands. I asked them to say their answers together. "How many crocodiles there are altogether!" I recorded this in the second column on the chart under the heading What We Are Trying to Find Out. The third column of the chart was labeled Special Things to Remember.

"Are there any things that we should remember when trying to solve this problem?" I asked. Their faces told me that they were perplexed by this question. I rephrased it to help them understand. "What do we know about counting and about this problem that will help us figure it out?"

Samantha chimed in, "We need to count them." I put Samantha's comment on the chart in the third column.

Abby added, "We need to remember that the numbers go in order from one to ten." I added Abby's idea beneath Samantha's.

I noticed Johnny looking at the first column of the chart, where the numbers of crocodiles were listed. He was trying to add the numbers in his head. I asked the children to turn to a partner and talk about ways they might solve this problem. Johnny waved his arm eagerly. "It's fifty-four," he proclaimed.

When I asked how he had figured the problem, he explained that he had kept adding on to the numbers from 1 to 10, adding 1 + 2 = 3, then 3 + 3 = 6, 6 + 4 = 10, and so on.

On a separate piece of chart paper, I recorded Johnny's strategy under the heading Strategies and asked students for other ideas. Garrett said to draw a picture of each crocodile. Chandler said to make a line of all the crocodiles and count them. Abby reaffirmed her previous idea of counting them in order from one to ten.

When no one else had ideas to share, I explained, "I have some Unifix cubes you may use to help you as you work with your partner to figure how many crocodiles. How might you use these cubes to help you?"

Isabella said, "Use Abby's idea of counting them from one to ten, but use each cube as one crocodile." I recorded Isabella's idea. As I passed out unlined paper, I explained that students were to work with their partner to solve this problem.

While the children worked, I noticed that the majority of them were quick to attempt both Abby's and Isabella's ideas. However, in their initial efforts, they naturally began working independently. I realized I needed to provide a quick demonstration of how to work together. With Austin's help, I modeled for the class talking together, working together, and taking turns. That seemed to do the trick, as only a few of the pairs needed further assistance with partnering.

Jordan and Anisha immediately put their cubes together in a pyramid fashion with one on top and ten on the bottom. They counted the cubes and easily came up with fifty-five. I asked them then to record their answer and what they did on the paper. Recording on paper was a challenge. They wrote *55* but were having trouble showing what they did to solve the problem. I asked if they could create the same picture on paper that they had made with the cubes. I hadn't anticipated the difficulty of this task for kindergartners.

Bryce, eager to use the cubes, began making long lines. "How many crocodiles have you counted so far?" I asked.

"I know there are forty-nine," he replied.

"How do you know?" I continued.

"I counted them while you were reading the book," he responded.

"How many do you have right now?" I asked.

"I don't know," he admitted.

I encouraged him, with his partner's help, to see if he could rearrange the cubes so that they showed how the crocodiles were introduced in the story. They made rows of cubes from one to ten. Both Bryce and Chase announced, "There are forty-eight!"

I realized they had left out the row of seven. I questioned, "Do you have a cube for every crocodile?"

"Yes!" they exclaimed.

"How do you know?" I asked.

They explained to me that they had counted. When I asked them to show me, they were able to see where they had missed a row of cubes.

"We can fix that," they both said confidently. They added the row of seven cubes, but they also had difficulty representing their thinking and problem solving on paper.

As a guest in Mrs. Fisher's class, I had about fifty minutes for this lesson; twenty of those minutes were set aside for the children to work. With few exceptions, the children were able to work with their partners and use the Unfix cubes to solve the problem. Only a handful of the thirty-three children in the class had difficulty counting the cubes (good information for further counting practice). When reflecting on this lesson, I decided my next visit should provide ample opportunities for the children to problem solve and represent their thinking on paper.

How to Get Students Talking!

Generating Math Talk That Supports Math Learning

By Lisa Ann de Garcia

Due to the attention in the last few years on discourse and its importance to student learning, educators nationwide are finding that they can help children become confident problem solvers by focusing on getting them to talk and communicate in partnerships, small groups, whole groups, and in writing. In addition, English Language Learners are flourishing as they experience focused opportunities for talking and trying on new mathematical vocabulary.

So what exactly is discourse? What are the teaching practices associated with successfully establishing an environment to support it, and as a result, to improve mathematical proficiency? How does one begin to elicit meaningful talk during math lessons? As a profession, we share a vision about the role student discourse has in the development of students' mathematical understanding, but are often slow to bring the students along. Children do not naturally engage in this level of talk.

This article addresses the above questions and concerns—and more. It opens with a look at discourse through NCTM's definition and its involvement with the Common Core State Standards. It then focuses on literature available on discourse, specifically the book *Classroom Discussions,* and addresses five teaching practices focused on the *how to* of getting students talking about mathematics. The article concludes with journaling insights on discourse from a kindergarten and second-grade classroom. This article is by no means an exhaustive list of discourse "to dos;" hopefully it will however get us all started in thinking about and implementing best talk practices.

What is Discourse in the Mathematics Classroom?

NCTM's Definition

The National Council of Teachers of Mathematics (NCTM) in their 1991 professional standards describes discourse as ways of representing, thinking, talking, agreeing, and disagreeing; the way ideas are exchanged and what the ideas entail; and as being shaped by the tasks in which students engage as well as by the nature of the learning environment.

A View Through The Common Core Lens

As much of the country begins to implement the new Common Core State Standards, it is important to reflect on the role of discourse in these new standards. The Common Core was created based on five process standards: communication, reasoning and proof (another form of communication), problem solving, representation, and connections. Evidence of the importance of communication in learning mathematics is found in the Common Core introduction in statements such as, "One hallmark of mathematical understanding is the ability to justify . . . a student who can explain the rule understands the mathematics and may have a better chance to succeed at a less familiar task . . ." (p. 4). In the grade-specific standards, the importance of communication in learning mathematics is reflected in statements such as , "Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense" (p. 33). These Common Core statements make it clear that conceptual understanding must be connected to the procedures, and that one way to deepen conceptual understanding is through the communication students have around concepts, strategies, and representations.

Learning from Literature on Discourse

One of the leading resources for discourse is *Classroom Discussions: Using Math Talk to Help Students Learn* (Chapin, O'Connor, and Anderson 2009). This resource and others highlight five teaching practices associated with improving the quality of discourse in the classroom.

Five Teaching Practices for Improving the Quality of Discourse in Mathematics Classrooms

1) Talk moves that engage students in discourse,
2) The art of questioning,
3) Using student thinking to propel discussions,
4) Setting up a supportive environment, and
5) Orchestrating the discourse.

Practice 1: Talk Moves That Engage Students in Discourse

For the first practice, the authors of *Classroom Discussions* propose five productive talk moves that can get talk going in an otherwise silent classroom. The first is *revoicing*. An example would be, "So you are saying that . . ." This *revoicing* allows the teacher to check in with a student about whether what the student said was correctly heard and interpreted by the teacher or another student. A way to encourage students to listen to their peers is through asking them to *restate someone else's reasoning*, such as, "Can you repeat what he just said in your own words?" Another way is to ask students to *apply their own reasoning to someone else's* using questions such as "What do you think about that?" and "Do you agree or disagree? Why?" This helps prevent students from just thinking about what they want to share and focuses their attention on what their classmates are saying. It also helps to strengthen the connections between ideas.

Simple questions such as, "Would someone like to add on?" are ways teachers can *prompt for further participation*. This helps elicit more discussion when not many students are talking, especially when they are not accustomed to explaining their thinking. Again it helps students to tune in to what others are saying so that they are able to expand on someone else's idea.

Perhaps the most valuable talk move suggested by Chapin, O'Connor, and Anderson is the use of *wait time*. Often teachers are too quick to answer their own questions when no one chimes in. Children quickly become accustomed to this. Waiting provides think time and sets the expectation that someone will indeed respond and that the teacher will wait until someone does. Another important use for wait time is to provide English Language Learners or anyone who needs extra time with an opportunity to process the question and formulate an answer. One teacher reported that in his initial uses of wait time, one of his English Language Learners was able to participate in class discussion for the first time.

Practice 2: The Art of Questioning

Questioning is another critical component in supporting students to engage in meaningful discussions. The NCTM Standards outline roles questions have in the math classroom. The first role, *helping students to work together to make sense of mathematics*, is addressed by the five talk moves discussed above. The second role, *helping students to rely more on themselves to determine whether something is mathematically correct,* can be supported by questions such as, "How did you reach that conclusion? Does that make sense? Can you make a model and show that?" Questions such as, "Does that always work? Is that true for all cases? Can you think of a counterexample? How could you prove that?" are designed to *help students to learn to reason mathematically*. To *help students to learn to conjecture, invent, and solve problems,* the teacher might ask, "What would happen if? Do you see a pattern? Can you predict the next one? What about the last one?" Finally, teachers use questions to *help students connect mathematics, its ideas and applications* by asking, "How does this relate to . . .? What ideas that we have learned were useful in solving this problem?"

Practice 3: Using Student Thinking to Propel Discussions

Because discussions help students to summarize and synthesize the mathematics they are learning, the use of student thinking is a critical element of mathematical discourse. When teachers help students build on their thinking through talk, misconceptions are made clearer to both teacher and student, and at the same time, conceptual and procedural knowledge deepens. When doing so, the teacher must be an active listener so she can make decisions that will facilitate that talk. She also needs to respond neutrally to errors, so that the students can figure out misconceptions themselves. For example, the teacher can ask the whole class, "What do you think about that?" when a student offers an incorrect strategy or can ask the rest of the class to prove whether or not the strategy works. Through the conversation, the misconception becomes apparent to the class. This practice results in an authentic discussion focused on the mathematics and not on the individual student. The teacher also needs to be strategic about who shares during the discussion, since it is not a show-and-tell session, and choose ideas, strategies, and representations in a purposeful way that enhances the quality of the discussion.

Practice 4: Setting Up a Supportive Environment

When setting up a discourse-rich environment and one that enhances student engagement, both the physical and emotional environment must be considered. Teachers who have studied engagement find that it is very effective if students face each other, either sitting in a circle or semi-circle on the floor or sitting in chairs arranged in a circle. Teachers can sit with students as part of the circle to encourage peer-to-peer discussion. If teachers are still having difficulty getting children to talk, they can remove themselves from the group and stand outside the circle. As a result, students are left looking only at each other, which encourages them to direct their comments to one another.

Careful consideration of the placement of visual aids and mathematically related vocabulary is important in supporting the level of talk. If charts are not visually accessible when they need to be, they will likely not be resourced by the students during whole group conversations. To increase the extent to which English Language Learners participate in group discussions, having related vocabulary and sentence frames where they can be easily accessed is critical.

For rich discussions, the emotional environment of the classroom must be safe and must be one where students want to learn and think deeply about the mathematics. When these elements are not present, the discussion stays at the surface level. Imagine a third grade classroom where the teacher introduces division for the first time and is met with cheers. It can happen! It happens when the value is on learning, challenging each other, and working together to solve problems as opposed to just getting the right answer. For more on setting up a supportive classroom environment for discourse, see Chapter 8 of *Classroom Discussions.*

Practice 5: Orchestrating the Discourse

The teacher becomes not unlike a conductor as he supports students to deepen their understanding of mathematics through a carefully orchestrated environment. In *Orchestrating Discussions,* Smith, Hughes, Engle, and Stein outline the *Five Practices Model*, which gives teachers influence over what is likely to happen in a discussion.

The Five Practices Model

The teacher's role is to:

1) anticipate student responses to challenging mathematical tasks;
2) monitor students' work on and engagement with the tasks;
3) select particular students to present their mathematical work;
4) sequence the student responses that will be displayed in specific order; and
5) connect different students' responses and connect the responses to key mathematical ideas.

Even if the teacher is focused, he still needs to hold students accountable. Otherwise the discussion will be unproductive. A lot of explicit teaching must go into how to engage in each level of discussion: whole group, small group, and partnerships. In the younger grades, one will find teachers showing students exactly what they should look like and sound like when discussing their thinking. Teachers may say things like, "Today in math, we are going to practice turning and talking with our partner. When I say go, you are going to turn like this and look at your partner. When I say stop, you are going to turn around and face me. Let's practice that right now." Even older students need to be explicitly taught what to do and say. A teacher might teach how a partnership functions by saying, "It sounds like you have an idea and you have an idea, but what seems to be lacking is for you two to put your ideas together to come up with a solution. So, what is your plan?"

One very effective method of holding students accountable is to let them know exactly what they should be saying when they are talking in their partnerships or small groups. For example, "Today, when you are talking to your partners and describing your solid shapes, I expect to hear you using the words faces, edges, and vertices." It is also supportive to let students know what they should be focusing on when someone is sharing a strategy, so they have a lens for listening, which heightens the level of engagement. A teacher might say, "When he is sharing his thinking, I want you to be thinking of how his way is similar or different to your way."

Students need to be aware of themselves as learners, and a great way to heighten this awareness is through self-evaluation and goal setting. Sometimes the child is the last one to know that he is distracting or not listening. Part of developing a safe culture is supporting students in being open with each other regarding their strengths and weaknesses so they can improve their communication skills and behaviors. It is wonderful to hear one child compliment another when she has participated for the first time or give gentle correction when another has been dominating the conversation. This level of self-awareness happens through consistent venues such as class meetings and tracking the progress of personal goals related to participation in mathematical discussions. The more students open up about themselves as learners, the deeper the relationships and, as a result, the deeper the trust.

Kindergarten						
Teaching Points	**Sept**	**Oct**	**Nov**	**Dec**	**Jan**	**Feb**
Partnerships						
Partner Talk Expectations	X					
Problem solving possible partner problems, such as: *"What do you do if you both want to go first?"* *"How do you talk to your partner if they are not sharing?"* Modeling language such as, "*You can go first, or I can go first"*		X				
"Turn and Talk", "hip to hip," "knee to knee"	X	X	much less prompting			
Demonstrating with a partner *Modeling with another student how to share*	X		X			
Showing Eye Contact	X	X	prompting	prompting		refine
What Listening Looks Like	X	X				
Teaching students to ask and answer a question on cue Ex: *"Turn and talk. First partner ask . . . second partner answer . . ."*		X				
Using partnerships to move towards whole group share of what they did together			X	X		
Comparing their work with a partner *Ex: Asking partner, "How did you sort?"* Partner answers, *"I sorted by . . ."*			X			
Have partners share in front of the whole group				X		
Introducing story problem procedures by saying the story a few times while students listen, then having them repeat it with the teacher a few times, then turn and tell their partner the story, then solve.				X		
Holding class meetings to help a partnership problem solve something related to working as partners						X
Formulating own question to ask their partner						X
Whole Group Discussion						
Comparing their work as a whole group *"Is what so and so did the same or different as what s/he did?"*	Very Guided					
Eye contact towards speaker		X				
Can you tell me what *so and so* said? (revoicing)		X				
"What do you notice about . . ." (this promoted a lot of talk)		X				
Learning to compare their work with others		X				
Prompting, *"Who is talking?" "What should you do?"*					X	
Turning and looking with just the heads and not entire bodies					X	
Whole group physical behaviors						X
Supporting Language and Vocabulary						
Use Sentence Stems *"When you turn and talk to your partner, I want you to use the words . . ."*	X	X	X	X	X	
Model Language*: "I say it, you say it."*	X	mimic with a partner	X			
Responding, *"I did it like so and so*"		X				
Language when comparing work: "*same/different, because*"		X	X	prompting		
Use of co-created charts / prompting students to reference them			X			
Vocabulary: agree/disagree			X			
Teaching how to ask a question back & generate own spontaneous questions					X	
Vocabulary: accurate / efficient						X exposure

Table 1: Teaching points of a kindergarten teacher during the year

2nd Grade						
Teaching Points	**Sept**	**Oct**	**Nov**	**Dec**	**Jan**	**Feb**
Partnerships						
Whisper to your partner (during whole group)	X					
"Did you and your partner agree or disagree?" (beginning listening and repeating	X					
Tell me what your partner said		X				
"You two don't agree? Who is right?" Don't just let it be, but push-back on each other		X Students are voicing disagreement on own respectfully	X	X		
"How can you figure that out?" "Can your partner help you with that?"		X				
Students are pushing on each other and keeping each other accountable		X				
Coaching on how to wait for your partner to finish her turn. *"Watch your partner." "Do you agree with how she took her turn?"*			X	Partner coaching really paying off!!		
Model how to help telling with out telling answer. *"You could say...you have a lot of coins, do you think you could trade?"*			X			
Disagreeing and justifying			X	X		
"Is the way he/she did it the same as how you did it?"			X	X		
Providing list of questions students were to ask as partnership during games (race to a stack with beans and cups)				X		
Talk to your partner about ____'s way				X		
Modeling how to ask partner to repeat and how to explain				X		
Using sentence starters				X		
Providing limited tools to promote discussion in small groups						X
Provide team activities where members have to decide how to solve and which strategy to share						X

Table 2: Teaching points of a second grade teacher during the year for Partnerships

2nd Grade						
Teaching Points	**Sept**	**Oct**	**Nov**	**Dec**	**Jan**	**Feb**
Whole Group Discussion						
Teach "quiet thumb"	X					
Respect: No laughing, mistakes are learning opportunities	X					
Good listening behaviors: No touching manipulatives, eye contact.	X	X			Reminders	
Physically adjusting eyes, heads, body	X				Reminders	
Begin number talks; collecting all answers without judgments	X					
Choosing kids to explain	X					
Ask questions to draw out solutions, such as, *"How did you figure that out?" "How did you count?" "Where did you start?" "Did you count like this or a different way?"* Modeling if they still cannot explain	X					
Strengthening listening by asking another child to repeat/explain strategy of another student	X	X				
Ask questions to hold students accountable for listening and deepening understanding such as *"Does that make sense?" "What do you think of what ____ said?" "Do you agree/disagree?" "Any questions for____?" "Who can explain ____'s strategy?" "What should you say if you didn't understand, couldn't hear, etc.?"*	X	X	X		Reminders	
Chart and name strategies students use, such as: *"Oh, you counted all, counted on, made a 10, used doubles."* Chart as the students talk to make steps visible.	X	X	X			
Referring to other kids' ways as a way to celebrate students taking risks by trying a new way	X	X	Kids starting to notice, "Oh, that is how __ did it"			
"Is your strategy the same or different than _____'s strategy?" "Which strategy did _____ use?" (referring to the chart)	X				X	
Teacher scripting children's strategies on their papers and on the chart.	X					
Highlighting students who try on another student's strategies		X				
Trying to get students to see that their peers are their teachers to foster reason for listening more carefully			X			
Getting students to try on another someone else's strategy and acknowledging it with students, such as *"Oh, Marquis did it like Yosef did yesterday."*			X			
Helping students learn how to articulate their thinking (e.g., *"What did you do? Tools you used? Where did you start?"*) to be easier understood by others				X		
Helping students to record their thinking. Model how to record each step so the listeners can see what you did				X		
Highlighting different ways of recording and different tools used in solving a problem (*"Let me show you another way to record" "When you put the blocks together, how can you show that on paper?"*)				X		
Slowing down the person sharing between each step and ask class *"Does that make sense?" "Do you understand" "Who can explain that step" "Why do you think she did that?"*				X	X	
"Which ways are the same or kind of the same?" "Who's might you try on?"				X		
Having preselected student writing strategies to share				X		
Discussing incorrect answers to see if kids will listen and respectfully agree and disagree				X		
Allow time for the other person to react to partner during share out				X		
Moving position from front of the room to promote explaining				X		
Share partner's strategy rather than your own					X	X
What do you think _______did next (heighten engagement)					X	
Using document camera more for share out since students have become more proficient with recording					X	X

Table 3: Teaching points of a second grade teacher during the year for Whole Group Discussion

First Discourse Experience 3rd- 6th Grade
Teaching Points

Whole Group Discussion

Explain that we are having a conversation about what we built (model for problem given)
What do we do when someone is explaining his/her thinking?
*Listen (not just hearing, but thinking about what they said)
*Listening to compare to see if we thought the same thing the speaker did
*What does paying attention look like?
Don't merely think what you are going to say next, rather respond back to the speaker - adding on or comparing
How do we talk like adults? - taking turns, not raising hands
Who would like to share? - opening it up to anyone (sometimes - other times choosing someone specific - this depends on if the focus is on the act of sharing or a specific strategy.
When one person shares, ask some to restate
Teach students how to ask someone to speak up or to repeat themselves if they weren't listening or if they couldn't hear "Could you please say that again, I wasn't listening."
Lots of turn and talk to partner with something specific to talk about
I have to listen so I can highlight a partnership and ask students to think about their thinking
Asking students to try on someone else's way and explain what they did.
Asking lots of questions such as "Does their way make sense?"
**It is necessary to remind students often where their eyes need to be and to listen to what the speaker is saying.

Partner Talk

Generally on the first day I go around and listen and make sure that the partnerships are working together rather than side-by-side play and coach accordingly
I will ask questions such as, "Do you know what he did?" "Can you explain it?"
Direct when necessary (if students are having trouble working together) by saying, "When we share out, I want you to explain what your partner did."

Note:

At the end of one lesson, the discourse is not beautiful, but if the teacher is explicit with expectations and how to engage in discourse. children will talk, mostly to partner, as they are a little shy about the group at first. Students definitely engage in what the other students are thinking and make sense of other strategies. I would expect to be emphasizing the above points repeatedly for the next couple of months.

Table 4: Teaching points that can be made on the first day in an upper grade classroom around discourse

Managing a classroom that makes students are responsible for their own learning means that the teacher has to become accustomed to not doing all of the work for them. One of the hardest things for teachers is to stop jumping in too soon and answering their own questions. Once a teacher I was working with told me that if she wasn't always doing the talking, she felt that she was not doing her job. Just because the students are the ones who should be doing the thinking and talking doesn't mean that the teacher does not play a significant role. One of the biggest jobs of the teacher is that of decision maker. The NCTM Standards state that teachers must decide what to pursue in depth, when and how to attach mathematical notation and language to students' ideas, when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with difficulty, and how to encourage each student to participate. These decisions, so well-articulated by NCTM, are central to effective math teaching and remain crucial as we move into the implementation of the Common Core State Standards for Mathematics.

A Look into Classrooms: Journaling About Discourse

Recently, a kindergarten and a second-grade teacher were invited to spend most of one school year journaling exactly what they do to explicitly teach meaningful mathematical discourse. I also reflected on what I do when I go into a 3rd – 6th grade classroom for the first time for a demonstration lesson and how I start to get students to talk when they are not accustomed to it. This analysis was further broken down into partnerships and whole group discussion. In the case of the kindergarten teacher, the explicit teaching she did to support language and vocabulary was also noted. The following tables outline the teaching points and what time of year each was a primary focus. For example, in kindergarten, the teacher worked on the children turning and talking in September and October. In November, much less prompting was needed, and after that it became a norm in the classroom culture.

Each group of students is unique and has different needs. The above insights are not meant to be a checklist or recipe of how to facilitate deep mathematical discourse in your individual classroom, but they can serve as a resource of the types of behaviors teachers need to explicitly teach and pay attention to when trying to deepen the quality of talk. They can also serve as a reminder that it is best to teach behaviors in small segments, especially with younger children. When teaching older children, unless they exhibit significant social difficulties, it may be possible to focus on several different aspects of talk at once, but these behaviors need to be reinforced on an ongoing basis. Once these behaviors become part of the classroom culture, it is important to refine and deepen the talk by addressing specific needs of the individual group of students.

Carrying Discourse into the Individual Classroom

Mathematics educators nationwide agree that student engagement in meaningful mathematical discourse has a positive effect on their mathematical understanding as they increase the connections between ideas and representations. As we begin to implement the new Common Core State Standards, we need to not only have a vision for what meaningful talk might look like, but also be equipped on how to get the talk going. Teachers need to explicitly teach the social behaviors necessary in engaging in discourse on a whole group, small group, and partnership level. Although there are common behaviors most teachers can initially address, most behaviors are unique to the dynamics of an individual classroom.

Works Cited

Chapin, S. H., C. O'Connor, and N.C. Anderson. *Classroom Discussions: Using Math Talk to Help Students Learn, Grades K-6, Second Edition* (Math Solutions, 2009)

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Recommended Reading List

Classroom Discussions: Using Math Talk to Help Students Learn, Grades K-6, Second Edition, S. H. Chapin, C. O'Connor, and N.C. Anderson.

Classroom Discussions: Seeing Math Discourse in Action, Grades K-6, N.C. Anderson, S.H. Chapin, C. O'Connor, (Copyright © 2011 by Scholastic, Inc.)

Good Questions for Math Teaching: Why Ask Them and What to Ask, K-6, Peter Sullivan and Pat Lilburn (Copyright © 2002 Math Solutions)

Good Questions for Math Teaching: Why Ask Them and What to Ask, Grades 5-8,
Lainie Schuster and Nancy C. Anderson (Copyright © 2005 Math Solutions)

I'm the Greatest!

Whole Number Directions (2 to 4 players)

1. In your journal or on a dry erase board, draw 9 lines like this:

   ___ ___ ___ , ___ ___ ___ , ___ ___ ___

2. Put the bean in the egg carton shaker and close the lid. Shake the carton and then open it.
3. Look under the bean. Write that number on any of the 9 lines.
4. Take turns shaking for numbers. Repeat until all 9 lines are filled. (You may not erase or change numbers after writing them down.)
5. Compare with the other players. Everyone must read their number aloud. The person with the largest number says "I'm the Greatest!"

I'm the Greatest!

Decimal Directions (2 to 4 players)

1. In your journal or on a dry erase board, draw 7 lines like this: ___ , ___ ___ ___ . ___ ___ ___
2. Put the bean in the egg carton shaker and close the lid. Shake the carton and then open it.
3. Look under the bean. Write that number on any of the 7 lines.
4. Take turns shaking for numbers. Repeat until all 7 lines are filled. (You may not erase or change numbers after writing them down.)
5. Compare with the other players. Everyone must read their number aloud. The person with the largest number says "I'm the Greatest!"

I'm the Greatest!

Teacher Directions and Preparation

Materials for each team:
1 egg carton, 1 bean or small object, paper and pencil or dry erase boards and markers

Preparation and Play:
1. Prepare the egg cartons by writing the numbers 0 to 9 in the bottom of each section of the cartons. You'll have to repeat two of the numbers.
2. Duplicate one of the two direction cards for each team (Whole Number or Decimal Directions). Fold the direction cards so that they stand up with the directions on one side and the title on the other.
3. Before distributing the egg cartons, model the game by playing it against the whole class a few times. To discourage cheating, make sure students keep their dry erase boards on top of their desks, visible to all. Remind them that after they write a number on a line, they may not change it.
4. After everyone is comfortable with the directions, distribute materials to teams for team play or place the game in a math center.

Variation:
Instead of using egg cartons, students may randomly choose a number in a different way. Options include drawing numbered cards from a deck or set of number tiles, spinning a spinner, or rolling a die.

Icebreakers
Group Profile
Materials: newsprint, markers, tape
Preparation: Trace an outline of the human body on newsprint. List the following topics outside the outline next to the coordinating body part:
Head: dreams or goals we have (for our community)
Ears: things we like to listen to
Eyes: How we like other people to see us
Shoulders: problems young people may have to face.
Hands: things we like to make or do (with our hands)
Stomach: things we like to eat
Heart: things we feel strongly about
Right foot: places we would like to go
Directions: Post outline of body on the wall. Invite participants to come up to the poster and write things or pictures to represent each area for them. This is done grafitti style, free form.
After everyone has had a chance to participate, ask for volunteers to report to the group on what is listed.
Discuss:

What are common interests? Shared goals? Dreams? Were there any themes? What are the things we feel strongly about? How do these relate to our group's work?

Incorporation
Explain that this game is about forming and reforming groups as quickly as possible. Don't worry if you are not even into the first group by the time the next group is called, just head to the next group. The idea is to meet many different groups of people as fast as possible. Get into a group of three...go!

Other suggestions:
A group of five with everyone having the same color eyes as you.
With the same last digit in their phone number as yours.
Wearing the same size shoe as you.
Get into a group of three people and make the letter "H" with your bodies.
Find everyone else born in the same month as you
Think of the first vowel in your first name, find four with the same vowel.

Sign Up Here

Materials: 6-10 pieces of large newsprint, tape, and pencils.

Preparation: Put pieces of the newsprint around the room. From the list of topics below, write a different topic of interest on the top of each newsprint. Also include a related question you want people to answer about each topic. (Topics can vary according to the age and interests of group involved):

I like to speak or perform in public. (What group(s) have you spoken to or performed in front of?)

I like to work on computers. (What programs do you know?)

I can speak a language other than English. (Which?)

I would be excited to travel in the U. S. or abroad. (Where? Where have you been?)

Making friends is an important part of my life. (Who are your best friends?)

My family is one of the things that makes me happy. (Something I like about them?)

There are things that I would like to change in this school. (What?)

There are things that I would like to change in our community. (What?)

The voting age should be moved from 18 to 21. (If you could vote, what law would vote to change?) I have organized or helped to organize an event, celebration, fund-raiser, meeting, wedding, or conference. (Describe.)

Instruct participants to walk around the room, look at the different topics and sign their name on any of the sheets that represent topics in which they have an interest, and to make a comment answering the question on each sheet.

After everyone has had a chance to sign the sheets, ask one person that has signed each sheet to read the names of the people that have signed that sheet and any comments.

Discussion: What interests does the group have? How many different interests are represented in the group? Which chart had the greatest interest? Which chart had the least interest? What does this say about the group as a whole? Is there a pattern? What comments are made?

Synthesis: Explain how these skills are important for community organizing and how each of them will contribute their interests and skills making the group stronger.

Who Am I?
The leader tapes the name of a famous person on the back of each participant. (i.e. Fred Flintstone, Mary Lou Retton, Bill Clinton, etc.) The group member is not to see who is taped to their back. Their task is to find out who they are. The participants go around the room asking others only yes or no questions. If the member receives a "yes" answer, they can continue to ask that person questions until they receive a "no" answer. Then they must continue on to ask questions to someone else. When a group member figures out who they are, they take off the tag, put it on the front of their shirt, and write their own name on it. That person can then help others find out who they are. The exercise concludes when everyone has discovered who they are.
Variation: Use names of famous pairs (like Syskell and Ebert, Bert and Ernie) and do a partner activity after the game.

Energizers
Chalkboard Sentences
Tell participants they will be competing to see which team is the first to complete a group sentence. Next, divide participants into two teams. If the group contains an uneven number, one person may compete twice. The leader sets up black-boards or newsprint for each team. The teams then line up 10 feet from their board. After giving the first person in each team’s line a piece of chalk or marker, explain the rules of the game. The rules are: Each team member needs to add one word to the sentence. Payers take turns; after they go to the board and write one word, they run back to give the next player the marker, and then go to the end of the line. (The sentence must contain the same number of words as there are members on the team.) A player may not add a word between words that have already been written. After, discuss the value of anticipatory thinking and the importance of individual cooperating in a group task).

2. **Spelling List of Names** - My first spelling list is made up of the names of everyone in the class. To make a game out of learning the names, I give each person an index card and pair the students randomly. The students interview each other and take notes about their partner's interests. Then we write down everyone's name on a large spelling list. While I'm writing the person's name, their partner stands up and introduces the student to the class. Students copy the names down and we use them in other activities like Bingo. I generally have them learn only the spellings of the first names, and if the class is large I divide the test into 2 different parts.

3. **Classmates Mix** - This is a fun icebreaker activity. Students mix around the classroom until you say give a signal to stop. Then they pair up with the closest person. You call out an icebreaker topic such as the ones below. Students talk over their answers until you call time, and then they begin mixing again. Continue with several rounds for as long as time allows. You might try 3 rounds one day and 3 rounds the next day if your students have trouble handling the movement at first. (The STOP technique works well for classroom management during classbuilders.) Here are some discussion topics to get you started:

- Share a little information about yourself and your family.
- What are some of your favorite things? Talk over your favorite foods, colors, animals, or anything else that's a favorite of yours.
- What do you like to do in your free time?
- What's the best book you have ever read? What did you like about it?
- What's the best movie you have ever seen? Why did you like it?
- What's your favorite subject in school? What do you like about this subject?
- What are your strengths? What kinds of things do you do well?
- How would you change this school if you were the principal?
- What can students do to make school a better place to be?

Classroom Discussions Using Math Talk in Elementary Classrooms

by Suzanne H. Chapin, Catherine O'Connor, and
Nancy Canavan Anderson

From Online Newsletter Issue Number 11, Fall 2003

Teachers typically are comfortable leading classroom discussions when teaching literature or providing social studies instruction. They value these discussions and rely on them to support students' learning. However, many teachers aren't as comfortable making use of classroom discussions for mathematics instruction. In Classroom Discussions: Using Math Talk to Help Students Learn, Grades 1–6 *(Math Solutions Publications, 2003), the authors present specific ways to lead classroom discussions that support students' mathematics learning and promote their ability to think, reason, and solve problems. In the book, the authors introduce five important talk moves, each a suggested action they have found to be effective in supporting mathematical thinking and learning. They illustrate each talk move with examples of actual classroom discussions and offer guidance to teachers for incorporating them into their own classroom instruction. Following is a brief introduction to two talk moves presented in the book—"revoicing" and "asking students to restate someone else's reasoning."*

Revoicing ("So you're saying that it's an odd number?")

When students talk about mathematics, it's often very difficult to understand what they say. Even if their reasoning is sound, it may not appear sound when they try to put their thoughts into words. Sometimes it's impossible to tell whether what they have said makes sense at all. And if you as the teacher have trouble understanding it, there's not much hope that the student's classmates will do any better. Yet given your goals to improve the mathematical thinking and reasoning of all students, you cannot give up on an especially unclear student. If the only students whose contributions are taken seriously are those who are easy to understand, few students will ever improve. Deep thinking and powerful reasoning do not always correlate with clear verbal expression.

Therefore, teachers need a talk move that can help them deal with the inevitable lack of clarity of many student contributions. They need a tool that will allow them to interact with the student in a way that will continue to involve the student in clarifying his or her own reasoning. And they need a tool that will help other students continue to follow along in the face of the confusion.

One such tool has been called "revoicing." In a revoicing move, the teacher essentially tries to repeat some or all of what the student has said and then asks the student to respond and verify whether or not the teacher's revoicing is correct, as in the dialogue below.

Ms. Davies has given her third graders a series of numbers and, in a whole-group discussion, has asked them to say whether the numbers are even or odd. They have established that if you can divide a number by two evenly, then it is an even number. Philipe has tackled the number 24. His contribution is less than completely clear.

1. Philipe: Well, if we could use three, then it could go into that, but three is odd. So then if it was . . . but . . . three is even. I mean odd. So if it's odd, then it's not even.

2. Ms. D: OK, let me see if I understand. So you're saying that twenty-four is an odd number?

3. Philipe: Yeah. Because three goes into it, because twenty-four divided by three is eight.

After hearing Philipe's confusing contribution, all Ms. Davies could grasp was that Philipe *might* be saying that 24 is odd. She hazards a guess in the form of a revoicing move: "So you're saying that twenty-four is an odd number?" By phrasing this guess as a question, she is essentially asking Philipe if her understanding is correct. By using this move, she gives him a chance to clarify. As it works out, he shows he did intend to claim that 24 is an odd number, and he gives his reason. By opening this conversational space for Philipe to respond, Ms. Davies has learned that he has a basic misconception about even and odd numbers. She has gained a foothold in the discussion that she did not have after simply hearing Philipe's first contribution.

While revoicing is especially useful in situations such as that described here with Philipe, it's also an effective move when you understand what a student has said but aren't sure that the other students in the class understand. Revoicing can make one student's idea available to others, give them time to hear it again, position a student's claim with respect to a previous student's claim in order to create the basis for an ongoing discussion, or focus on a change that has occurred in the discussion. Revoicing provides more thinking space and can help all students track what is going on mathematically.

Asking students to restate someone else's reasoning ("Can you repeat what he just said in your own words?")

In the previous example, the revoicing move was used by the teacher. However, the teacher can also extend the move to students, by asking one student to repeat or rephrase what another student has said and then immediately following up with the first student. Ms. Davies used this move to continue the classroom conversation.

4. Ms. D: Can anyone repeat what Philipe just said in his or her own words? Miranda?

5. Miranda: Um, I think I can. I think he said that twenty-four is odd, because it can be divided by three.

6. Ms. D: Is that right, Philipe? Is that what you said?

7. Philipe: Yes.

This move has several potential benefits. First, it gives the rest of the class another rendition of the first student's contribution. It gives them more time to process Philipe's statement and adds to the likelihood that they will follow the conversation and understand his point. It thereby supports the teacher's goal of full access to participation. This move is particularly valuable for students whose first language is not English. Second, this move provides evidence that the other students could and did hear what Philipe said. This is important: if students could not or did not hear what a speaker said, they cannot easily participate in further exchanges. Finally, it yet again clarifies the claim that Philipe is making and provides Philipe with evidence that his thinking is being taken seriously. Over time, as students come to realize that people are listening closely to what they say, they increasingly make efforts to make their contributions comprehensible.

Math Talk and Helping Community in *Math Expressions*

Dr. Karen Fuson

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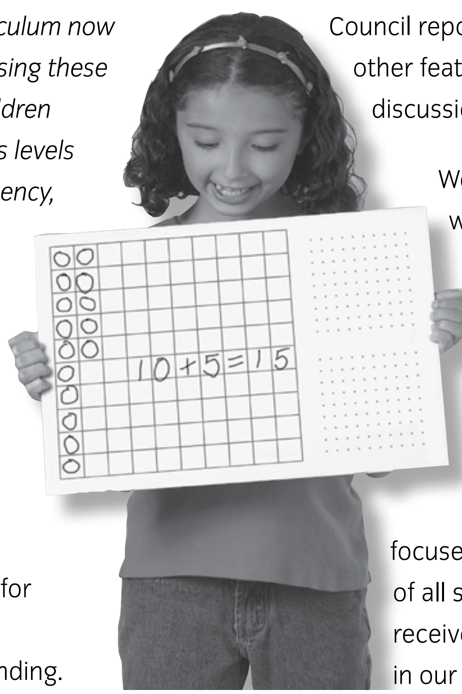
This is the second article describing the classroom structures found in the **Math Expressions** *program. These structures are Building Concepts, Math Talk, Student Leaders, Quick Practice, and Helping Community. Though we discuss the five structures in separate author papers, they interact synergistically in the classroom. The* Children's Math Worlds Research Project (CMW) *that developed the curriculum now called* **Math Expressions** *found that using these structures in the classroom enables children from all backgrounds to learn ambitious levels of mathematics with understanding, fluency, and confidence.*

WHAT IS MATH TALK?

The National Council for Teachers of Mathematics (NCTM) Standards and two recent National Research Council reports *(Adding It Up: Helping Children Learn Mathematics* and *How Students Learn: Mathematics in the Classroom)* emphasize the need for students to discuss their mathematical thinking as a way to increase understanding.



The CMW Research Project that developed the ***Math Expressions*** program undertook research to identify crucial aspects of such discussion and to identify levels through which teachers could move from traditional teacher-focused teaching to productive student-to-student discussion monitored and supported by the teacher, as recommended by the National Resource Council reports. We also sought to identify other features that could support effective discussions.

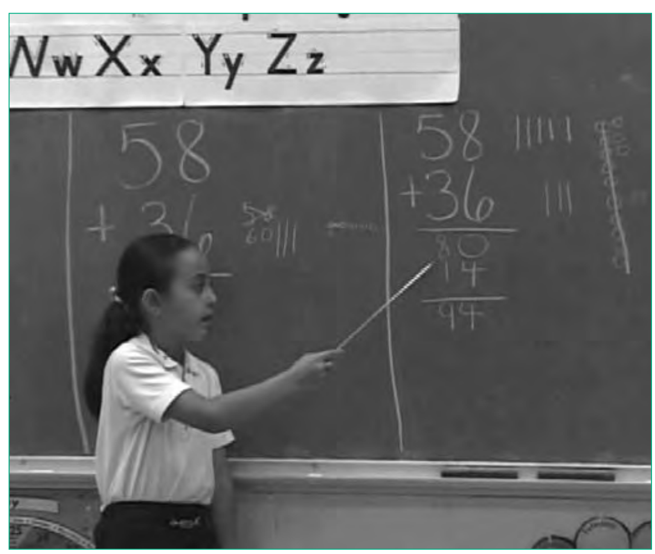
We found that the term *Math Talk* was effective in communicating with teachers and students the focus on discussion we desired. Our view was that Math Talk is an instructional conversation directed by the teacher but with as much direct student-to-student talk as possible. Math Talk is focused on developing the understanding of all students in the class. However, we received reports of teachers not involved in our CMW project who were using Math

Talk in other ways. Some teachers just had students take turns telling about their problem-solving methods or participate in undirected talk rather than actually analyzing and comparing methods. Some teachers did not seem to be trying to help students move along a learning path toward more effective methods of problem solving.

In true Math Talk, teachers create a Math Talk Inquiry environment and encourage constructive discussion of problem-solving methods through well-defined classroom activity structures, based on the four crucial aspects of Math Talk. Table 1 (see insert) describes the four aspects of Math Talk (questioning, explaining math thinking, source of math ideas, and responsibility for learning), describes the levels of Math Talk, and summarizes roles of the teacher and of the students in the highest level, Level 3. This table was developed initially from a case study of a teacher who moved from Level 1 to Level 3 with the support of a researcher. Then the table was modified as we studied other teachers moving through the levels and was published as a research paper (Hufferd-Ackles, Fuson, & Sherin, 2004). By now many teachers have used the table to help them move through the levels. There are also many Math Talk boxes in the ***Math Expressions*** Teacher's Editions that provide specific help in doing Math Talk.

LEARNING MATH TALK

Effective Math Talk cannot be implemented into a classroom overnight. A teacher must work his or her students up to Level 3 Math Talk over time. It often takes two or three months to build a classroom up to Level 3 if students are not familiar with Math Talk from the start. Initially the teacher and more advanced students will do a lot of modeling of solving and explaining for other students. In the beginning, teachers also concentrate on building listening skills by asking students to repeat a problem, question, or explanation in their own words. Teachers also elicit questions from students. These may apply to any topic (e.g., *How does your method relate to the method that Sam just explained?* or *Why did you do this method?*) or may apply specifically to a given math topic. In the latter case, the teacher needs to model some of these questions to use new math vocabulary, though often more advanced students can also think of such questions. Students need to learn to stand beside their drawing and numerical work and point to parts of it with a pointer as they explain. Students often initially explain only one part of their thinking or explain it incompletely. Using questions can help expand a student's explanation. The teacher or another student may also expand or clarify the student's explanation



through questions, though always checking with the original explainer to be sure that the assistance is what the student meant.

In addition to helping students learn Math Talk methods, teachers often need to adjust to the Math Talk structures themselves. They must learn to wait patiently and use a “bite your tongue” strategy to allow student talk to emerge. They must also physically move to the side or back of the room to facilitate student-to-student talk so that the explainer looks at classmates and not at the teacher. From the side or back of the room, a small gesture can be used to remind the explainer to look at classmates rather than at the teacher. Teachers can provide community assistance by asking explainers if they need help, but they also need to allow wait time before doing so. Shy students initially may need the presence of a friend at the board with them even if the friend does not help with the explaining. As teachers provide the space and support for students’ voices to emerge, they often report being frequently amazed by the mathematical thinking their students are able to express.

ENGAGING IN MATH TALK

We found that two kinds of Solve, Explain, Question, and Justify classroom activity structures were very effective in engaging all students in Math Talk. In both structures, all students solve problems simultaneously. In the first structure, as many students as possible go to the board to solve a problem while the rest of the students work at their seats. Then the teacher selects two or three students from the board who have interesting solutions, or need the chance to explain their work, to talk about their solution. Only two or three students need to explain their work since usually students cannot maintain concentration for more than two or three discussions of the same problem. Next a different group of students goes to the board to solve the next problem. This process is very motivating. Most students enjoy solving problems at the board even if they do not get the chance to explain their work. While the students are working at the board, the teacher has a chance to see how solutions evolve. The teacher also gets a good feeling for how individual students are doing. In one class period many or even all of the students can get a turn at the board.

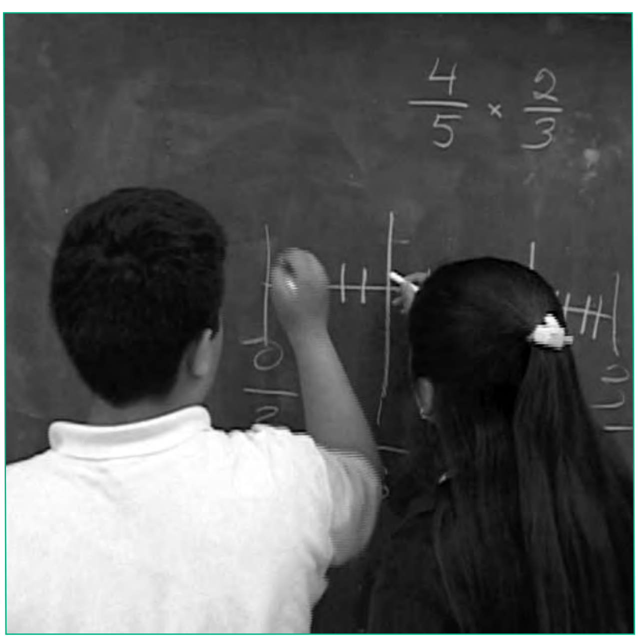
The second effective classroom structure allows every student in the classroom to explain his or her solution. Every student solves a problem at his or her seat. Then two or three students are selected by the teacher to go to the board to draw their solutions. The students left at their desks then pair up and explain their solutions to each other. Then the class discusses the solutions of those students at the board. Students at their seats can write their solutions on paper, which can be picked up and skimmed later by the teacher to see how students are doing. Another option is to have students at their desks solve problems on the large (12” by 19”), individual ***Math Expressions*** dry-erase boards called MathBoards. These MathBoards permit the teacher to send any additional student to the board for another explanation of the discussed problem because the drawings on the MathBoard are large enough to be seen by classmates.

An important feature of both of these classroom structures is that no class learning time is lost. In other approaches, when students are sent to the board to draw their work, the rest of the class remains at their seats doing nothing. In the case of these ***Math Expressions*** structures, the students at their desks are just as involved in the problem solving as those at the board. Sometimes a step-by-step variation of these activity structures is helpful. Teachers can have each student explain one step of a solution at a time until a final solution is reached. Another method is to put students in pairs. The pairs can solve together and explain their work, with the less-advanced student explaining first and the other, more-advanced student expanding and clarifying as needed.

The opportunity for all students to explain their math thinking over time is especially valuable for students learning English, as well as for native speakers advancing their verbal communication skills. Ultimately, developing understanding and verbal communication will aid all students in their future education and careers. In addition to verbal communication, the use of math drawings is central to Math Talk. Math drawings can show the quantities in a computation and relate them to a written numerical method or can show the situation in a word problem. The math drawings help everyone understand the student's math thinking. The special learning supports on the MathBoards enable students to learn meaningful drawings rapidly and then the open space on the MathBoards is used for math drawings.

A NURTURING MEANING-MAKING MATH TALK COMMUNITY

The Math Talk Inquiry environment is also a Nurturing Meaning-Making Helping Community in which everyone is a teacher and a learner. This creates a secure base for learning and for Math Talk. It enhances everyone's mathematical understanding, competence, and confidence. Teachers build the Helping Community daily by helping students learn how to help each other at the board, in pairs or groups, or when working individually. ***Math Expressions*** was developed through 10 years of intensive classroom research in many classrooms with students from many different cultural and linguistic backgrounds. The Nurturing Meaning-Making Math Talk Helping Community enables students from all backgrounds to bring their family and cultural experiences into the classroom and be validated, affirmed, and understood.



The Math Talk approach used by ***Math Expressions*** supports deeper understanding and more complex language learning than other reform approaches because the ***Math Expressions*** program provides research-based learning paths that move all students forward. Students develop the prerequisite understandings so that they can invent interesting methods. Research-based accessible strategies are taught so that everyone has an effective method. These relate to common methods so that all methods can be discussed and related. Many students learn, relate, and explain multiple methods. Math drawings enable students to solve and explain more effectively and enable listeners to understand and question more effectively. This process of discussing the whole developmental range of solution methods permits differentiated instruction to occur in whole-class activities but also enables students to move forward to a mathematically-desirable effective method. ***Math Expressions*** truly supports teachers as they develop a Nurturing Meaning-Making Math Talk Inquiry Classroom.

<table>
<tr><th colspan="4">Levels of Math Talk Learning Community: Teacher and Student Action Trajectories</th></tr>
<tr><th colspan="4">Components of the Math Talk Learning Community</th></tr>
<tr><th>A. Questioning</th><th>B. Explaining math thinking</th><th>C. Source of math ideas</th><th>D. Responsibility for learning</th></tr>
<tr><td colspan="4">Overview of shift over Levels 0–3: The classroom community grows to support students acting in central or leading roles, and shifts from a focus on answers to a focus on mathematical thinking.</td></tr>
<tr><td>There is a shift from the teacher as questioner to the students and teacher as questioners.</td><td>The students increasingly explain and articulate their math ideas.</td><td>There is a shift from the teacher as the source of all math ideas to students' ideas also influencing the direction of lessons.</td><td>The students increasingly take responsibility for learning and evaluation of others and of themselves. Math sense becomes the criterion for evaluation.</td></tr>
<tr><td colspan="4">Level 0: This is a traditional teacher-directed classroom with brief answer responses from students.</td></tr>
<tr><td colspan="4">Level 1: The teacher is beginning to pursue student mathematical thinking. The teacher plays a central role in the Math Talk community.</td></tr>
<tr><td colspan="4">Level 2: The teacher models and helps students build new roles. Some co-teaching and co-learning begins as student-to-student talk increases. The teacher physically moves to the side or back of the room and directs from there.</td></tr>
<tr><td colspan="4">Level 3: The teacher is a co-teacher and co-learner. The teacher monitors all that occurs and is still fully engaged. The teacher is ready to assist, but now in a more peripheral and monitoring role (coach and assister).</td></tr>
<tr><td>The teacher expects students to ask one another questions about their work. The teacher's questions still may guide the discourse.

Student-to-student talk is student-initiated, not dependent on the teacher. Students ask questions of each other and listen to responses. Many questions are "Why?" questions that require justification from the person answering. Students repeat their own or other students' questions until they are satisfied with the answers.</td><td>The teacher follows along closely to student descriptions of their thinking, encouraging students to make their explanations more complete; he or she may ask probing questions to make explanations more complete. The teacher stimulates students to think more deeply about strategies.

The students describe more complete strategies; they defend and justify their answers with little prompting from the teacher. The students realize that other students will ask them questions, so they are motivated and careful to be thorough. Other students provide support with active listening.</td><td>The teacher allows for contributions from students during his or her explanations; he or she lets the students explain and "own" new strategies. The teacher is still engaged and deciding what is important to continue exploring. The teacher uses student ideas and methods as the basis for lessons or mini-extensions.

The students contribute their ideas as the teacher or other students are teaching, confident that their ideas are valued. The students spontaneously compare and contrast and build on ideas. Student ideas form part of the content of many math lessons.</td><td>The teacher expects students to be responsible for co-evaluation of everyone's work and thinking. He or she supports students as they help one another sort out misconceptions. He or she helps and/or follows up when needed.

The students listen to understand, then initiate clarifying other students' work and ideas for themselves and for others during whole-class discussions as well as in small group and pair work. The students assist each other in understanding and correcting errors.</td></tr>
</table>

Table 1

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Math Talk

Algebra Summit
SAS Institute
August 12, 2010
Presented by Michelle Tuck-Thomas

Frayer Foldable

- Fold a piece of the lavender paper in half length-wise.
- Fold it again width-wise.
- Grab the vertex on the folds and fold it such that a triangle is created-Some of us know this as “dog ears.”
- Unfold.

Frayer Foldable

- Upper left: Definition
- Upper right: Environment/Transformation
- Lower left: How Do I Get There?
- Lower right: For My Classroom

Math Talk

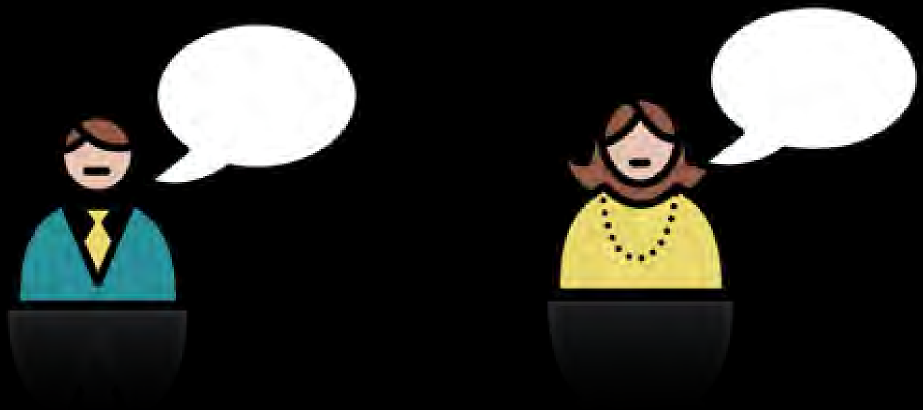
- Define “Math Talk” in your own words. Take a few minutes to reflect and write down your thoughts.

Math Talk

- Math Talk is not just simply having students take turns telling their method.
- It is an instructional conversation directed by the teacher that allows students to talk as much as possible.
- Math Talk is focused on developing understanding for all children in the class.

Math Talk

- So, what does an environment conducive for “Math Talk” look like?
- Talk with a partner and brainstorm a few ideas.



Math Talk Environment

- In true Math Talk, teachers create a Math Talk Inquiry environment and encourage constructive discussion of problem-solving methods through well-defined classroom activity structures, based on the four crucial aspects of Math Talk.

Dr. Karen Fuson, *Math Expressions*

Math Talk Environment

- Gradual process
- Requires teachers and students taking on new roles
- Teachers must work their way up all three levels
- Everyone is a teacher and a learner

Math Talk Key Components

- Questioning
- Explaining Math Thinking
- Source of Math Ideas
- Responsibility for Learning

Levels of Math Talk

- Level 0: This is a traditional teacher-directed classroom with brief answer responses from students.
- Level 1: The teacher is beginning to pursue student mathematical thinking. The teacher plays a central role in the Math Talk community.

Levels of Math Talk

- Level 2: The teacher models and helps students build new roles. Some co-teaching and co-learning begins as student-to-student talk increases. The teacher physically moves to the side or back of the room and directs from there.

Levels of Math Talk

- Level 3: The teacher is a co-teacher and co-learner. The teacher monitors all that occurs and is still fully engaged. The teacher is ready to assist, but now in a more peripheral and monitoring role (coach and assister).

Dr. Karen Fuson, *Math Expressions*

- **Levels of Math Talk Learning Community: Teacher and Student Action Trajectories**
- Let’s take a few minutes to read over this.

Gradual Transformation

- A shift from teacher as sole questioner to both children and teacher as questioners
- Children increasingly explaining and articulating their math ideas
- A shift from teacher as the source of all math ideas to children's ideas also influencing the direction of lessons

Gradual Transformation

- Children increasingly taking responsibility for learning and for the evaluation of themselves and others
- Increasing amounts of student-to-student talk with teacher guidance as needed

Math Talk Learning Community

- How might we go about transforming our classrooms into student-centered, student-led “Math Talk” environments?
- Find your **quadrant partner #2** and discuss this.
- Record your responses on the chart paper provided.

Math Talk Structure

- Solve and Discuss
- Step-by-Step
- Student Pairs
- Student Leaders
- Scenarios
- Small Groups

Math Talk Structure

- What other ideas do you have to engage learners in “Math Talk?”
- Let’s share aloud so that everyone benefits from our discussion.

Algebra Talk

- A class conversation about an algebraic equation, in which students discuss and critique various strategies for solving the problem.
- The work is done mentally, though some writing may be offered by a student or a teacher when a strategy is explained.

Algebra Talk Rules

- When the problem is displayed, solve it in your head.
- When you have solved the equation, put your thumb up in front of your chest.
- Try to solve it in a different way.
- For each way you solve, put up another finger.

Algebra Talk: One Step Equations

- $X + 2 = 6$
- $3 = y + (-7)$
- $a – 3 = -2$
- $b – (-8) = -2$

Algebra Talk: Two Step Equations

- $2x + 1 = 5$
- $3x – 1 = 5$
- $2 – 4x = 6$
- $13 = 7x - 1$

- Pair up with your **Quadrant Partner # 4**.
- Read the problems found in the plastic bags.
- Discuss how you might engage students in "Math Talk" as part of your instructional activities.

What Are My Next Steps?

- Take a few minutes to reflect on what you will do to implement "Math Talk" in your classroom?
- Record your responses on your foldable.

Resources can be found at:

- http://www.eduplace.com/math/mthexp/pdf/mathtalk.pdf
- http://www.edugains.ca/resources/frameworks/MathTalkLearningCommunityResearchSynopsis.pdf
- http://web3.d25.k12.id.us/PDF/Curric/math_expressions-math_talk.pdf

Math Talk Learning Community

What Is Math Talk?

The **NCTM Standards** emphasize the importance of developing mathematical language and communication in order to understand concepts rather than merely following a sequence of procedures. *Math Expressions* seeks to build a community of learners who have frequent opportunities to explain their mathematical thinking through Math Talk and thereby develop their understanding. children are asked to solve problems, explain their solutions, answer questions, and justify their answers. They use proof drawings as a reference for their explanations.

The dialogue that takes place helps everyone understand math concepts more deeply, and it helps children to increase their competence in using mathematical and everyday language. While children engage in dialogue, the teacher acts as a guide to maintain the focus of the discussion and to clarify when necessary.

Math Talk

Math Talk is not just taking turns telling your method or meandering undirected talk. It is an instructional conversation directed by the teacher but with as much direct child-to-child talk as possible. Math Talk is focused on developing understanding for all children in the class.

Multiple Benefits

Children gain greater understanding and ownership of mathematical concepts as they develop and express their own ideas. Describing one's methods to others can clarify one's own thinking. Similarly, hearing and analyzing others' approaches can supply one with new perspectives; and frequent exposure to different approaches engenders flexible thinking. Math Talk provides opportunities for children to understand errors they have made and permits teachers to assess children' understanding on an ongoing basis. By building understanding, Math Talk also prepares children for taking tests. When children encounter complex problems in testing, they can rely on their knowledge of the underlying mathematical concepts, developed through Math Talk activities, to successfully unravel and solve the problems. Math Talk also helps with test items that require explaining an answer.

What Is a Math Talk Learning Community? How Do You Build One?

Children use discussion to support the mathematical learning of everyone in the class. In a mature community of this type, children:

- develop and share their own mathematical thinking
- listen carefully to the mathematical ideas of others and restate them in their own words
- ask questions about and provide insights into the mathematical ideas of others

A Gradual Process. The development of such a community is a gradual process that may take several months. The classroom is transformed as children and teacher take on new roles and responsibilities in a variety of areas.

At the beginning of this process, teachers model Math Talk for children and elicit responses. Teachers wait patiently and refrain from intervening immediately to correct children's errors in order to create space and support for children's voices to emerge. Teachers eventually guide children from the side or the back of the classroom so that children can sense that their questions, ideas, and discoveries are the focal point of instruction.

Elements of the gradual transformation include:

- a shift from teacher as sole questioner to both children and teacher as questioners
- children increasingly explaining and articulating their math ideas
- a shift from teacher as the source of all math ideas to children's ideas also influencing the direction of lessons
- children increasingly taking responsibility for learning and for the evaluation of themselves and others
- increasing amounts of child-to-child talk with teacher guidance as needed

My Notes

Within such a community, being able to use appropriate math vocabulary, language, and proof drawings helps math become personally meaningful to children and provides a context through which children can share their ideas.

Extending Math Talk

- Solve and Discuss
- Step-by-Step
- Student Pairs
- Student Leaders
- Scenarios
- Small Groups

Structures for Developing Math Talk Skills. The key supports for Math Talk are the various "participant structures," or ways of organizing class members as they interact. You can easily familiarize yourself with the most common Math Talk structures described in *Math Expressions:*

- **Solve and Discuss (Solve, Explain, Question, Justify):** Have four to five children go to the board, and each child solves the problem, using any method he or she chooses. Their classmates work on the same problem at their desks with paper or MathBoards. Then ask two or three children at the board to explain their methods. Children at their desks are encouraged to ask questions and to assist each other in understanding the problem and solution.
- **Step-by-Step:** In this variation of the "Solve and Discuss" method, several children go to the board. This time, however, different children perform each step of the solution, describing the step before everyone else does it. Children at the board and at their desks carry out that step.
- **Student Pairs:** Two children work together to solve a problem, explain a solution method to each other, role-play within a mathematical situation, play a math game, or help a partner having difficulties.
- **Whole-Class Practice and Student Leaders:** In *Math Expressions* children develop into leaders with Quick Practice activities. You can blend this strategy into your daily instruction. Initially children who understand a concept and are beginning to achieve speed and fluency lead the class; eventually everyone is a Student Leader.
- **Scenarios:** The main purpose of scenarios is to demonstrate a mathematical relationship in a visual and memorable way. In a scenario, a group of children is called to the front of the classroom to act out a particular situation.

- **Small Groups:** You can encourage unstructured groups to form spontaneously if there is room. For some activities, you may prefer to assign children to specific groups. When children finish, the groups present results, with each member explaining one part of the solution or project.

A full description of and instructions for introducing each of these approaches can be found in the *Math Expressions* Teacher Guide. When working on the Level 1 blending of *Houghton Mifflin Math* and *Math Expressions*, you may want to introduce several of the participant structures early in the year. In this way, you will familiarize children with Math Talk and begin to develop a Math Talk learning community in your classroom.

More or Less?

A Lesson with Kindergartners and First and Second Graders

by Rusty Bresser and Caren Holtzman

featured in *Math Solutions Online Newsletter*, Spring 2007, Issue 25

The concepts of more, less, *and* the same *are basic relationships contributing to the overall concept of number. In this activity, students gain experience with these concepts and are also asked to think about part-part-whole relationships. (For example, students see that when twenty two-color counters are tossed, a variety of combinations of yellow and red can result: ten red and ten yellow, nine red and eleven yellow, and so on.)* More or Less? *is excerpted from Rusty Bresser and Caren Holtzman's* Mini-lessons for Math Practice, Grades K–2 *(Math Solutions Publications, 2006), a book that provides multiple opportunities for students to practice and deepen their understanding of concepts they have learned.*

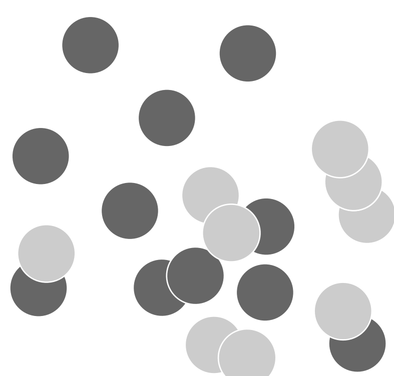
Elisabeth Frausto held up a paper cup, showing it to her students, who were assembled in a circle on the rug. Inside the cup were twenty counters; one side of each counter was red and the other side was yellow.

"There are twenty counters inside the cup today," Elisabeth began. She held up one of the counters, showing both sides. The class had experienced this activity for several months, first working with ten counters in the cup, then fifteen, and now twenty counters.

"I'm going to spill the counters onto the rug. I want you to think about whether there are more yellows, more reds, or about the same amount of reds and yellows."

As Elisabeth shook the cup, the students wiggled with excitement.

"Are you ready?" Elisabeth asked. "Show me you're ready." She waited until the students seemed calm and were sitting on the rug with their hands folded. She then spilled the counters onto the rug.



"Are there more red counters, more yellow counters, or about the same amount of each color?" Elisabeth asked.

The students in the circle offered a mix of opinions; some thought there were more yellow, some thought there were more red, and a few thought there were about the same amount of each color.

After eliciting students' ideas, Elisabeth sorted the reds and yellows into two groups, being careful not to flip any counters. She once again asked the students whether they thought there were more yellow or more red counters. This time, a majority of students thought there were more red counters.

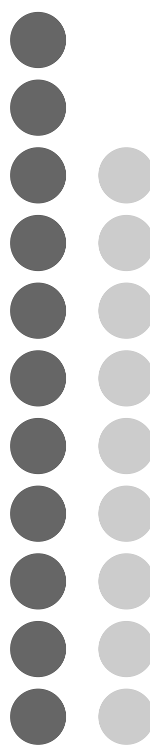
"How can we find out whether there are more red counters or more yellow counters?" Elisabeth asked the class.

"Count them!" several students chorused.

"And match them up, one to one!" others advised.

The students knew that Elisabeth would match up the counters one to one because they'd seen her do it previously as part of the activity.

She then picked up one counter at a time and positioned a red next to a yellow until there were no more yellow counters left, leaving two extra red counters.



"Are there more reds or yellows?" Elisabeth asked the class.

"More reds!" several students chorused.

"There are less yellows!" other students chimed in.

"Yes, there are fewer yellows," Elisabeth paraphrased. She didn't want to correct the students' language but rather to model the correct language.

Together with the students, Elisabeth counted the number of yellow counters. Once it was confirmed that there were nine yellows, Elisabeth asked the class how many red counters there were.

"There's eleven reds," Miles said.

"How did you figure it out?" Elisabeth asked.

"'Cause there are two more red than yellow. There are nine yellow and two more red than that, so that's eleven."

"So how many red counters and yellow counters altogether?" Elisabeth asked.

There were a few confused looks, but most students responded that there were twenty counters in all.

"There's twenty 'cause we started with twenty!" Sophie exclaimed.

Even though students had had previous experience with this activity using ten and fifteen counters, Elisabeth knew that some students would need more experience with the activity to realize that the total amount stays the same each time.

"How many reds are there?" Elisabeth asked again.

"Eleven!" students chorused.

"And how many yellows?"

"Nine!"

"What's eleven and nine more?" Elisabeth pressed.

"Twenty!" the class called back.

To finish the activity, Elisabeth recorded the equation on the board that represented the combination of twenty counters: *11 + 9 = 20.* She then repeated the activity one more time, putting the twenty counters back into the cup, shaking it, and spilling the counters onto the rug. This time, there were seven reds and thirteen yellows, a different number combination of twenty for students to work with.

Extending the Activity

- Use a different number of two-color counters (if you started with ten or fifteen, use twenty or twenty-five counters; if you started with twenty, use ten, fifteen, or twenty-five counters).
- Have students partner up, giving each pair a cup and twenty two-color counters. This way, students gain direct experience with the activity. Have students record the outcome of each spill by drawing a picture of the counters or writing an equation.

More than One

A Lesson for First and Second Graders

By Jamee Petersen

From Online Newsletter Issue Number 16, Winter 2004–2005

Miriam Schlein's More than One *(New York: Greenwillow, 1996) introduces the concept that one unit can be made up of more than one thing. One pair is always two, one week is seven days, but one family can be two, three, or more people.*

Here, first and second graders use mathematical language to describe their self-portraits, applying the concept that one can be an exact number or can describe a variable number of things. This lesson appears in the new book Math and Nonfiction, Grades K–2, *by Jamee Petersen (Sausalito, CA: Math Solutions Publications, 2004).*

I gathered the students on the rug to read *More than One* aloud to them. When I finished reading, I asked, "What did you learn about the number one from this book?"

"One can mean one like one sun in the sky, but sometimes it means more than one," Brock offered.

"One can mean a really big number like a billion or trillion," Amanda stressed, "like sand. One beach has so much sand we can't even count it all, but all the sand put together is just one beach."

"One can mean nine. Like in baseball, nine players make a team. But one team can also mean five, like a basketball team, and a football team has eleven players on the field at once," explained Mark.

"So, one can refer to one exact thing, but one can also refer to groups that have more than just one in them," I said. On the board I wrote *More than one* and under it I created a two-column chart, titling the columns *exact numbers* and *variable numbers.* Although *variable* wasn't a term that the students were familiar with, I introduced it by simply using it throughout the lesson along with more familiar language like the word *changeable.*

"What were some of the ideas presented in the book about the number one that we could list in our exact numbers column?" I asked.

Meagan said, "*Pair* would go there."

"You're right," I confirmed. I recorded the word *pair* under the heading exact numbers and asked, "Can you explain why, Meagan?"

"Well, like a pair of shoes is two, but it's one pair of shoes," she said.

"One dozen always means twelve, like a dozen doughnuts," said Jasmine. I added *dozen* to the first column in the list.

"One week is always seven days, so *week* should go under exact numbers," Sari directed. I wrote *week* in the first column.

"I think *team* would go there even though it changes because it's still an exact number," Eric said. I understood what Eric was trying to explain and saw that a few other classmates shared his reasoning. I asked Sammy to explain further.

"It's like one team is an exact number, like nine players on a baseball team," she shared. "But if you're talking about another team, like a volleyball team, then the number would be six, so one team is six players."

"I think *team* would go on the *various* side because it is like family; it depends on what kind," Julian said. I didn't correct Julian's mispronouncing *variable. Various* is a word that Julian was familiar with, and I felt confident that he would make the shift after hearing *variable* used more often.

Others agreed with Julian, so I wrote *team* under both the exact numbers and variable numbers columns.

"How was the number one used in the book to describe groups with changing numbers?" I asked and recorded as the children reported. Soon our list looked like this:

More than one . . .

exact numbers	variable numbers
pair	team
dozen	family
week	flock
team	ocean
	forest
	beach
	crowd
	school

Then I introduced the activity and said, "So the book *More than One* showed us how one can mean more than one. For example, we are one class, but within this class are twenty-six students. I'd like you to begin thinking about the one *you* that helps make up our one class. Today we are going to create self-portraits and describe them using some of the mathematical ideas presented in the book. For example, let's focus on our faces."

I asked the students to turn so they were knee-to-knee with the person sitting next to them. "Study your partner's face. How many eyes do they have?"

“Two,” chorused the class.

“How many *pairs* of eyes do they have?” I emphasized by pointing to *pair* on our chart.

“One,” the students replied.

I continued inquiring, asking about the number of ears and number of pairs of ears, and then about noses and nostrils, mouths and lips.

I moved on to include the variable numbers. “So we have many pairs on our face—a pair of eyes, ears, nostrils, and lips. Now everyone smile a big toothy smile at your partner. How many smiles do you each have?”

“One,” the students replied, giggling.

“How many teeth are in that smile? I want you each to count your own teeth using your tongue and then open up and have your partner check your count just like a dentist might look into your mouth.” The students were a little silly during this activity but it was worthwhile because it generated a set of numbers that varied because some of their smiles were missing teeth.

Giovanna said, “I have a smile with nineteen and a half teeth because I lost this one and now I have a big tooth coming in, but it is only halfway in. So I have nineteen teeth plus this half tooth.”

“So we each have one mouth or smile with one set of teeth, but the number of teeth that makes up that smile or set can be different. Some of you have twenty-four teeth, some have twenty-five and a half, and others have twenty-six. The number of teeth varies,” I said, pointing to the heading *variable numbers* on the board, “but the number of mouths on each face is still just one.”

We talked about the tops of our heads and the hairs on them being too many to count, like the trees in the forest and the birds in the flock we had seen in the book’s illustrations.

I asked the students to turn and face me. I told them again that they were going to be drawing portraits of themselves. “Then you’ll write at least five mathematical statements that describe your face,” I said. For those students who needed an extra mathematical challenge or who enjoyed drawing, I suggested, “Or you can draw your entire body and include additional facts about one part. For example, you have one hand but five fingers.”

I made mirrors available to students—a full-length mirror and several hand mirrors. Some students studied themselves carefully with the mirrors before they got started and as they drew. Others drew their portraits without so much as a glance in a mirror.

A sentence starter is always helpful for some students. I listed some prompts on the board for them to record on their papers and think about:

I have two _____ but just one _____.

I have five _____ but just one _____.

I have lots of _____ but just one _____.

The idea of "more than one" is depicted in Figures 1 and 2. Joe's and Morgan's drawings both show eyelashes. Each chose to handle this number differently. Joe used an exact number, *4,* and then was sure that his drawing matched that number. Morgan was comfortable using an approximation, stating that she had *lots of eay lashes,* a statement that accurately describes her drawing.

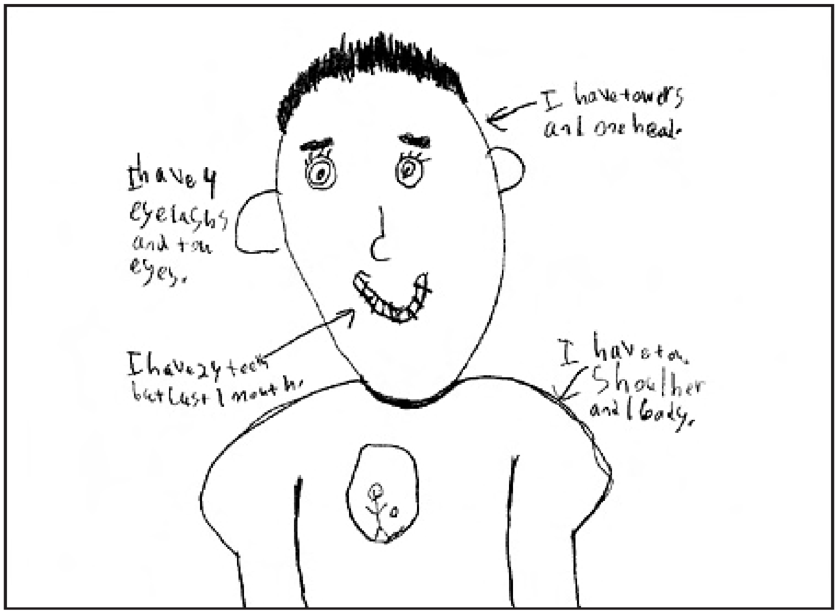


Figure 1. Joe worked diligently to complete his illustration. The concept of "more than one" came easily to him, but expressing it in writing proved more challenging.

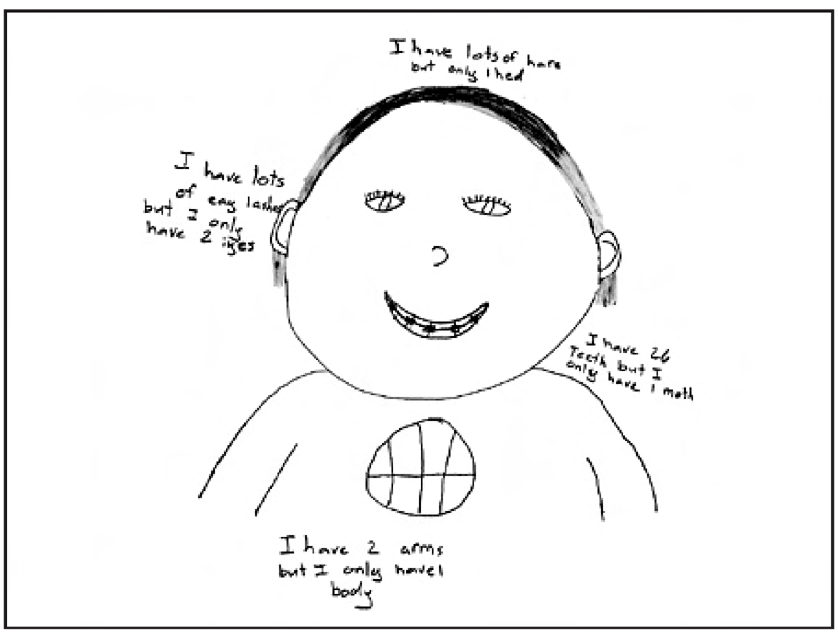


Figure 2. Morgan's labels surrounded her self-portrait.

After the students had completed their self-portraits, I collected the pictures and hung them in the hallway. Next to them I posted a sheet of paper on which I had recorded two questions taken from the book, leaving space beneath each question to record answers.

Can one be more than one?

Can one be different, different every time?

The following day I asked the students to talk about their drawings and what they wrote. I asked them to answer the first question I had written: "Can one be more than one?"

"Yes," they replied together. I wrote *Yes.*

I read the next question: "Can one be different, different every time?"

Again the students replied easily: "Yes."

I wrote: *Yes, one can be twenty-six kids in a class. How many are in your class?* I hoped this last question would prompt readers of our display to think about the mathematical idea being presented. Over the next week, many classes stopped outside of our room to answer the questions and take note of the mathematical concepts the students had illustrated in their drawings.

Number Talks: Thinking with Numbers
By Kathy Richardson

What young children know and understand can never be fully determined through paper and pencil tasks. Teachers can get much more complete and useful information if they watch and interact with the children while they are doing mathematical tasks. Number Talks are one such way to interact with the children. How the children respond reveals their level of understanding.

Number Talks provide opportunities for children to work with computation in meaningful ways. During Number Talks, the teacher presents various problems to groups of children and asks them to share the processes they used to figure out "how many." Number Talks can be held either with the whole class or with small groups. When children are working with the whole class, they will have opportunities to experience a wide range of problems and many different ways to solve them. When working with a small group, the teacher can make sure all the children have the opportunity to share their processes if they wish, and can more easily tailor the problems to meet the needs of a particular group.

Helpful Hints for Implementing Number Talks

1. Do number talks every day but for only 10 minutes. A few minutes more often is better than a lot of minutes infrequently.
2. Ask questions such as...
    - How did you think about that?
    - How did you figure it out?
    - What did you do next?
    - Why did you do that? Tell me more.
    - Who would like to share their thinking?
    - Did someone solve it a different way?
    - Who else started the problem this way?
    - Who else used this strategy to solve the problem?
    - What strategies do you see being used?
    - Which strategies seem to be efficient, quick, simple?
3. Experiment with using the overhead, the whiteboard, chart paper, etc.
4. Consider having students "circle up" in chairs or on the floor.
5. Give yourself time to learn how to...
    - record student solutions
    - listen to and observe students
    - collect notes about student strategies and understanding
6. To help determine what numbers or problems you select use what you learn from previous number talks as well as the focus of your daily classroom instruction.
7. Do number talks with yourself and others to try new strategies and increase your own confidence.
8. Name/label the strategies that emerge from your students:
    - Use doubles
    - Break apart numbers
    - Make it simpler
    - Use landmark numbers (25, 50, 75, 200, etc.)
    - Use a model to help
    - Use what you already know
    - Make a "10"
    - Start with the 10's
    - Think about multiples
    - Think about money
    - Traditional algorithm
    - Counting on
9. Use related problems: 3 x 14, 3 x 114, 3 x 1014 or 7 + 8, 27 + 8, 107 + 8 or 3 x 7, 6 x 7
10. Do number talks in small groups
11. Ask students to "Do as much of the problem as you can."
12. Give students lots of practice with the same kinds of problems.
13. Use numbers for subtraction and addition that require students to work past a ten or hundred. Example: 56 + 7 = 87 - 9 = 25 + 6 = 94 + 8 = 106 - 8 =
14. Give students opportunities to add and subtract 9, then 8 etc., using 10 as a friendly number to work with. Example: 68 + 10 = 78 so 68 + 9 = 77

15. Expect students to break apart numbers, not count on their fingers. Show them how. 6 + 8 (think of 6 as 4 + 2; add the 2 to 8 to get 10 and just add the remaining 4 to get 14)
16. Show the strategy you used. Make sure they know it's not "the" way, just another strategy.
17. Give students larger numbers so they can give "estimates."
18. If you use chart paper, write down the student's name next to their solution. Keep track of who is participating and their strategies. Use the following as a "sorting" or assessment guide:
    - Can figure it out (by counting on, using an involved strategy etc.)
    - Beginning to use efficient strategies (can complete some of the problem efficiently)
    - Just knows or is using efficient strategies
19. Create a safe environment. When children feel safe, they are comfortable sharing an answer even when it's different from everyone else's.
20. Provide concrete models (snap cube "trains", base 10 blocks, money etc.)
21. Give opportunities for children to "think first" and then check with the models.
22. Have students occasionally record their thinking and the steps they use to solve a problem.
23. Encourage self-correction; it's okay to change your mind, analyze your mistake, and try again.
24. Provide number stories.
25. Be curious; avoid making assumptions.
26. Give number talks time to become part of your classroom culture. Expect them to follow the usual learning curve stages. "Keep on keeping on" and you will get positive results!

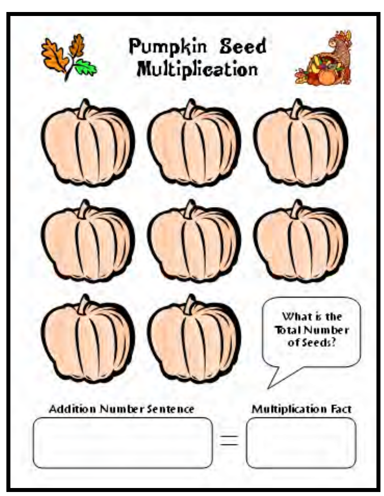
Pumpkin Seed Multiplication

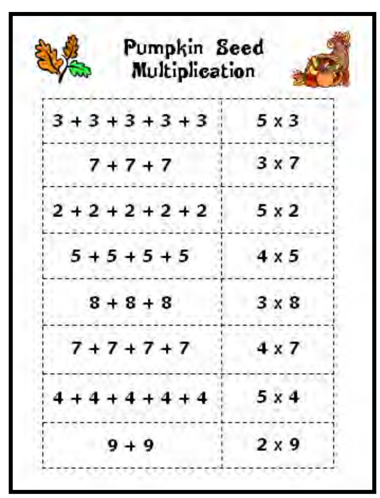
Teaching Suggestions

Pumpkin Seed Multiplication is a hands-on activity to help students make the transition from addition to multiplication. For each pair of students, you'll need a game board, a set of directions, a set of Addition Sentence cards, a set of Multiplication Fact cards, and a small bowl of about 35 "pumpkin seeds." You can use real pumpkin seeds (dried, of course!) or any small item such as dried beans, paperclips, plastic chips, or unit cubes. If you would like to create additional number sentences and multiplication facts, use the blank cards on the last page.

Before introducing the game, make sure students have had other opportunities to explore the meaning of multiplication and how it relates to addition. They should have already been introduced to the idea that a multiplication fact such as "3 x 5" means "3 groups of 5" and is equal to "5 + 5 + 5." Start off by choosing a student to help you model the game in front of the class. Review the directions on the next page and demonstrate how to do each step. Note that this is a learning game, not a competitive game, and there are no winners and losers. Then assign partners and hand out the materials, or allow students to play the game in a math center. The game does not require students to write down the number sentences and total seeds, but you can certainly have them do that in a math journal.

Note: This lesson is adapted from Fishbowl Multiplication, an activity in Mastering Math Facts - Multiplication and Division, Laura's next Power Pack!





Pumpkin Seed Multiplication

3 + 3 + 3 + 3 + 3	5 x 3
7 + 7 + 7	3 x 7
2 + 2 + 2 + 2 + 2	5 x 2
5 + 5 + 5 + 5	4 x 5
8 + 8 + 8	3 x 8
7 + 7 + 7 + 7	4 x 7
4 + 4 + 4 + 4 + 4	5 x 4
9 + 9	2 x 9

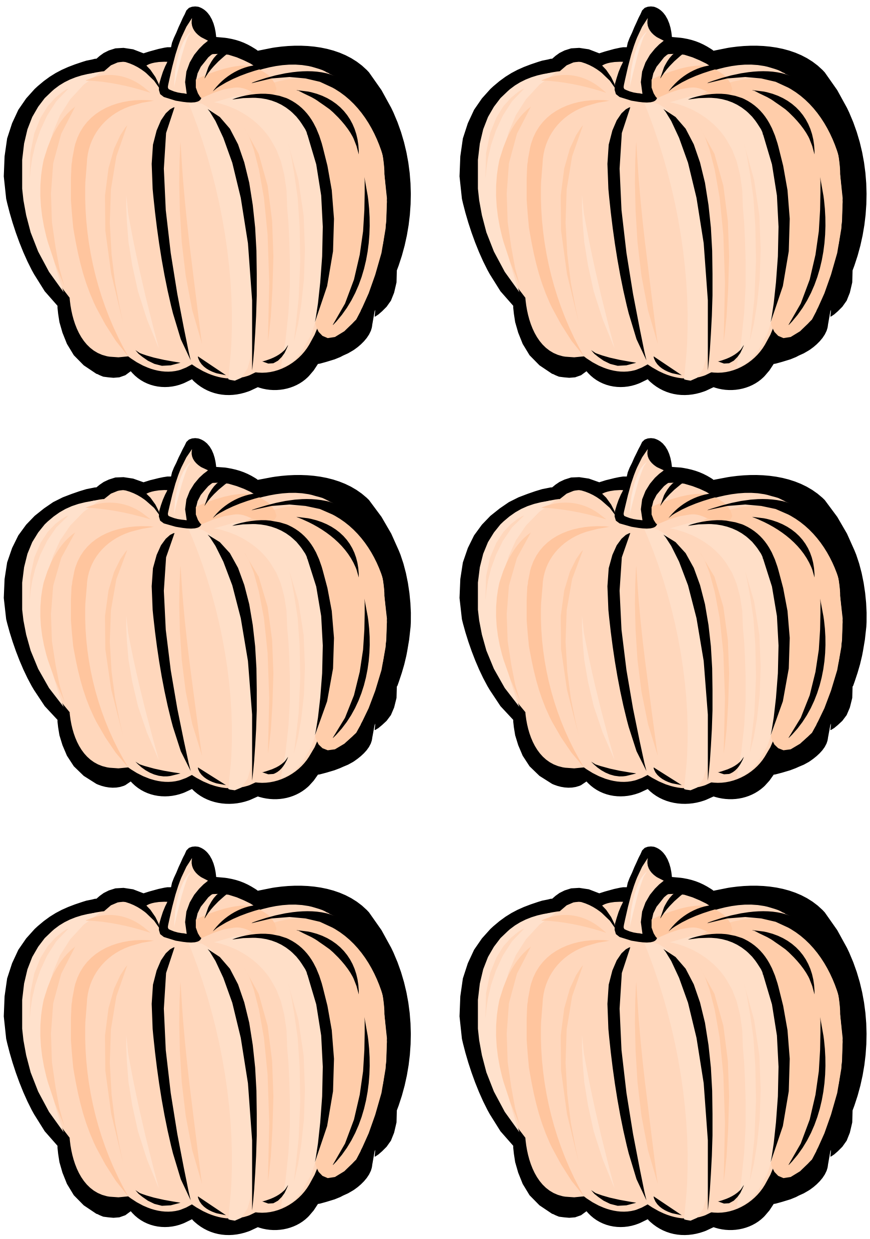
Pumpkin Seed Multiplication

Number of Players: 2
Materials: Pumpkin Gameboard
Addition Number Sentences and Multiplication Facts
Bowl with 35 Seeds (paperclips or other small objects)

Directions:

1. Cut apart the Addition Sentence and Multiplication Fact cards and separate them. Place the Addition Sentences face down in a pile. Spread the Multiplication Facts out on the table, face up.
2. Decide who is Partner A and who is Partner B. Partner A chooses an Addition Number Sentence and places it face up in the box at the bottom of the page. He or she uses the numbers on the card to fill the pumpkins with equal groups of seeds.
3. Partner B finds the matching Multiplication Fact and places it face up in the other box. Partner B then counts to find the total number of seeds and announces it to Partner A.
4. If Partner A agrees, remove both cards and set them aside. Put all the seeds back in the bowl. If not, recount the seeds.
5. Repeat steps 1 - 4, with Partner B choosing the Addition Sentence and Partner A finding the Multiplication Fact and counting the seeds.
6. Continue taking turns until all the cards have been used or the time runs out.

Pumpkin Seed Multiplication



What is the Total Number of Seeds?

Addition Number Sentence = **Multiplication Fact**

Pumpkin Seed Multiplication

3 + 3 + 3 + 3 + 3	5 x 3
7 + 7 + 7	3 x 7
2 + 2 + 2 + 2 + 2	5 x 2
5 + 5 + 5 + 5	4 x 5
8 + 8 + 8	3 x 8
7 + 7 + 7 + 7	4 x 7
4 + 4 + 4 + 4 + 4	5 x 4
9 + 9	2 x 9

Pumpkin Seed Multiplication

Team Talk Question Cards

What is your favorite subject? Why do you like it?	What do you like to do in your free time? Why do you like this activity?
What is the best book you have ever read? What did you like about this book?	What would you like to be in the future? How could you go about reaching that goal?
Where would you go on vacation if you could go anywhere? Why?	If you could have just one wish, what would it be and why?
Share a little information about yourself and your family.	What qualities do you look for in a friend? Why are these things important to you?

Question Cards

Topic:

MAKING SENSE

Teaching and Learning Mathematics with Understanding

James Hiebert
Thomas P. Carpenter
Elizabeth Fennema
Karen C. Fuson
Diana Wearne
Hanlie Murray
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Foreword by Mary M. Lindquist

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2 The Nature of Classroom Tasks

One of the most important points that we will make is that students develop mathematical understanding as they invent and examine methods for solving mathematical problems. This is quite different than the usual claim which says that students acquire understanding as they listen to clear explanations by the teacher and watch the teacher demonstrate how to solve problems. In this chapter, we will explain what we mean when we say that students should be encouraged to invent and examine methods for solving problems, and we will show why this is essential for building important mathematical understandings.

Why Are Tasks Important?

Students learn from the kind of work they do during class, and the tasks they are asked to complete determines the kind of work they do (Doyle 1983, 1988). If they spend most of their time practicing paper-and-pencil skills on sets of worksheet exercises, they are likely to become faster at executing these skills. If they spend most of their time watching the teacher demonstrate methods for solving special kinds of problems, they are likely to become better at imitating these methods on similar problems. If they spend most of their time reflecting on the way things work, on how various ideas and procedures are the same or different, on how what they already know relates to the situations they encounter, they are likely to build new relationships. That is, they are likely to construct new understandings. How they spend their time is determined by the tasks that they are asked to complete. The tasks make all the difference.

Students also form their perceptions of what a subject is all about from the kinds of tasks they do. If they are asked in history class only to memorize the names, dates, and locations of historical events, they will think that history is about remembering facts from the past. If students are asked in mathematics class only to practice prescribed procedures by completing sets of exercises, they will think that mathematics is about following directions to move symbols around as quickly as possible. If we want students to think that doing mathematics means solving problems, they will need to spend most of their time solving problems. Students' perceptions of the subject are built from the kind of work they do, not from the exhortations of the teacher. These perceptions guide their expectations for what they will do in mathematics class and influence their inclination to participate in the kind of classroom community we are describing in this book (see Chapter 4). Once again, it starts with the tasks. The tasks are critical.

What Kinds of Tasks Are Important?

What kinds of tasks should teachers use if they want their students to build important mathematical understandings? As we argued in Chapter 1, students build mathematical understandings by reflecting and communicating, so the tasks must allow and encourage these processes. This requires several things: First, the tasks must allow the students to treat the situations as problematic, as something they need to think about rather than as a prescription they need to follow. Second, what is problematic about the task should be the mathematics rather than other aspects of the situation. Finally, in order for students to work seriously on the task, it must offer students the chance to use skills and knowledge they already possess. Tasks that fit these criteria are tasks that can leave behind something of mathematical value for students. We will explore these criteria for selecting and designing tasks in the next section.

Tasks Should Encourage Reflection and Communication

Reflecting and communicating are the processes through which understanding develops. One of the simplest principles we can suggest is that if you would like students to understand, then be sure they are reflecting on what they are doing and communicating about it to others. Tasks are the key. They provide the context in which students can reflect on and communicate about mathematics.

Reflecting means turning something over in your head, thinking again about it, trying to relate it to something else you know. If a task encourages you to reflect on something, you do not rush through it as quickly as you can. Tasks that encourage reflection take time. Communicating means talking and listening. It means sharing the method you developed to solve a problem and responding to questions about your method. It means listening to others share their methods and asking questions to make sure you understand.

In order for students to reflect on mathematics and communicate their experience, they must see that there is something intriguing on which to reflect and something worthwhile to communicate. They must sense a difficulty that they would like to resolve and discuss. In order for the task to meet these needs, two things are essential: one is that students must make the task their own. Students must set the goal of solving the problem. The second is that the intriguing or perplexing part of the situation should be the *mathematics*. The task could, of course, be interesting in lots of ways, but if students are to build *mathematical* understandings, then it should be interesting in a mathematical way.

For something to be a problem for a student, he or she must see it as a challenge and must want to know the answer. The student must set a goal of resolving the problem. The goal might come from the student, or be adopted by the student after listening to peers or the teacher. The important thing is that the student makes the goal his or her own.

Goals come in many shapes and sizes: A goal might be to find the answer to a question posed by the teacher or by peers; it might be to increase the efficiency of a computation procedure; it might be to develop a strategy to solve a large, messy problem; it might be to generate a problem for others to solve. All of these goals define problems for students. They set intellectual challenges that create the need for resolution. The goals might set short-term problems, solvable in a few minutes, or they might set long-term, large-scale problems, solvable only after days or weeks.

Students will work to achieve goals only if they believe the goals are worth the effort. The reasons for perceiving worth may include the student's personal values (remember that all students are naturally curious) or values emerging in the culture of the classroom (for example, students may wish to participate in the class discussion and have something to contribute; see Chapters 3 and 4). It is important that the student attaches worth to the goal beyond that of immediate external rewards (Hatano 1988). If students are working only toward an external reward, such as leaving for recess early, this can work against reflecting thoughtfully about what they are doing.

Some tasks suggest interesting problematic situations that are not very mathematical. For example, suppose a group of sixth graders was given a budget of $100 and asked to plan a class party. This kind of task has become a favorite mathematical activity. However, the task may be resolved with little mathematics, especially of a kind that would challenge sixth graders. The situation still might be problematic, but the problems raised and resolved might be social or political ones.

The reverse can also occur. Some situations might look mathematical but would not be very problematic. For example, suppose a student, say Joanne, wanted to memorize the multiplication facts, perhaps to please her parents, or her teacher, or even herself. She borrowed a set of flash cards from her teacher and drilled herself for a number of days until she knew them all. Although we might applaud Joanne's motivation and discipline, and agree that knowing multiplication facts is important, we would not say that she engaged in solving a problem. The activity might even be called mathematical, but it did not make mathematics problematic. There was no need to search for and develop a method to solve the problem, and there was no need to reflect on what was happening. This is important because it means that it is unlikely that the activity facilitated the development of Joanne's mathematical understanding. We are not saying the activity was wasteful or unimportant: The point is that understanding develops only as students reflect on and communicate about situations that are mathematically problematic.

Here is an example of a task that is mathematically problematic: Suppose students are presented with the task of developing a method for adding $\frac{1}{3} + \frac{1}{4}$. If students have not yet added fractions with unlike denominators, this could be a task that allows the mathematics to become problematic for students. They would need to rely on their past experiences and then extend their knowledge to generate a solution. They would need to reflect on what they know about fractions, perhaps use resources such as diagrams or fraction pieces, and continually think about whether the methods they are developing are consistent with methods for related problems and whether the answer that is produced is reasonable. All of this involves intensive reflection. If students are asked to work on the task in small groups or to present and defend their method of solution, then communication becomes an integral part of the activity. Communication increases the likelihood that students will think again about their own method, and hear about other methods that would work just as well or better. It is not hard to see that understanding would be a natural outcome of this kind of task.

Tasks Should Allow Students to Use Tools

Tasks that encourage reflection and communication are tasks that link up with students' thinking. One way to describe this is to say that students should see ways in which they can use the tools they possess to begin the task. We define tools broadly to include things the student already knows and materials that can be used to solve problems. Tools are resources or *learning supports,* as we will call them in Chapter 5, and include skills that have been acquired (e.g., counting and adding single-digit numbers can be used as tools to add multidigit numbers), physical materials (e.g., fraction pieces can be used to add $\frac{1}{3} + \frac{1}{4}$), written symbols (often used as records for things that have been figured out), and verbal language (often used to communicate with others about the task). We will describe the role of tools more completely in Chapter 5, so we will make only a few points here, in relation to tasks.

Using tools to work on mathematical tasks can be thought of like using tools to complete tasks around the home. Tools are very handy, and we use many of them without even thinking. We use our reading skill to study the directions for how to open the new aspirin bottle; we use water, detergent, and a dish cloth to wash the dishes; and we use our fingers to flip the latch on the window. How did we learn to use these tools so well? We learned to use them because we were given time to explore the tools and time to practice using them in different ways. Of course, it is likely that we also had a bit of instruction in how to use them, but we did not learn by sitting back and watching someone else use them. One does not learn to use a hammer skillfully by watching someone else hammer nails. Tools are used effectively when their owners can practice using them on a variety of tasks. It is the same way with mathematical tools. Students need to have time to explore them, try them out, and use them in a variety of situations.

A second thing about tools is that they are used when there is a need to use them, when they can help to solve a problem or complete a task. Tools are used for a purpose. It is likely that you did not practice using a dish cloth just so you could get good at it. There were dirty dishes that needed to be washed. The same is true for mathematical tools. Students get good at using mathematical tools by using them to solve problems. Usually there is little point in practicing with tools just to be practicing.

It is important to note that tools are used when the *user* sees a need for using them. This means two things: One is that the user chooses the tool to use and finds out if it was a good choice by using it. Choosing a sledge hammer rather than a nail hammer to pound in the tomato stake may not be the best choice (the sledge hammer may be too heavy for the thin stake) but the user will learn about sledge hammers and nail hammers and tomato stakes by trying it out. Something similar is true for mathematical tools. Counting by ones may not be the best tool to find 45 + 38, but the tool user will find out something valuable about counting by ones and about 45 + 38 by trying it out, reflecting on the process, and communicating with others about it.

Another implication of using tools when the user sees a need is that the tasks need to be suitable for the tools that are available. It would be inappropriate to ask someone to build an intricate piece of furniture if they had never used a saw or chisel. Building a wood crate for storing toys might be a better first task. Similarly, it would be inappropriate to ask students to solve $\frac{1}{4} + \frac{1}{3}$ if they did not yet know the meaning of $\frac{1}{4}$ and $\frac{1}{3}$. This does not mean that tasks should be easy for students, or that students should know how to complete them before they start. Rather, it

We cannot provide a list of all the residues that are important because there is no one correct list, and if there were it would be very long. There are many kinds of understandings that are important, and different students are likely to build different ones. We can, however, identify two types of residue that are essential and that can provide useful guides for selecting tasks. One type can be called insights into the structure of mathematics, and the second type is the strategies or methods for solving problems.

Mathematical systems are filled with relationships. Take the base-ten number system for an example. The simple looking numeral 328 is loaded with relationships that can be constructed by students—relationships between the values of the digits, between the units represented by the different positions, and so on. Tasks that invite students to explore relationships of this kind, while they are solving problems, are likely to leave behind insights into the structure of this mathematical system (Cobb et al. 1991; Fuson and Briars 1990; Hiebert and Wearne 1993).

Tasks that are likely to focus students' attention on mathematical relationships are tasks such as: developing several different methods for solving 28×17 and discussing the efficiency of the methods; finding how many triangles can be drawn inside a rectangle, pentagon, hexagon, and so on, using the vertices of the polygon, and looking for a pattern; and deciding whether it is possible to find a fraction between any two fractions and explaining why or why not. Tasks like this provide opportunities for students to get inside mathematical systems and discover how they work. In general, tasks that encourage students to reflect on mathematical relationships are likely to leave behind insights into structure.

If tasks are problematic for students, and if students are allowed to work out methods to complete the tasks, then they also are likely to take with them strategies for solving problems. Two kinds of strategies will be left as residue. One kind of strategy is a specific technique for completing specific kinds of tasks. Two quite different examples will help to illustrate this process. First, consider a routine-looking computation problem. Suppose students had not yet added decimal fractions and the task involved adding $1.34 + 2.5$. After students developed methods for completing the task they would likely take with them specific strategies that could be used to add similar decimal fractions in the future.

A second example comes from a larger scale real-life situation. The day we were writing the first draft of this chapter, Cal Ripken Jr. broke Lou Gehrig's record for consecutive games played in Major League baseball. Mr. Gehrig's record was 2130; Mr. Ripken was playing in his 2131st consecutive game. The Baltimore Orioles had especially large crowds means that students should already have some tools available that allow them to begin thinking about the problem and trying out methods that might work. After students have talked about fractions such as $\frac{1}{4}$ and $\frac{1}{3}$, and perhaps represented them with fraction pieces, then they have some tools they can use to begin solving $\frac{1}{4} + \frac{1}{3}$. Tasks should be challenges for students, but they should link up with where students are and with what they already know and can do.

Tasks Should Leave Behind Important Residue

William Brownell (1946) pointed out a number of years ago that it is better to think of understanding as that which comes naturally while students solve mathematical problems rather than as something we should teach directly. More recently, Davis (1992) suggested that we have too long been designing our curriculum and instruction on the idea that we should first teach students skills and then have students apply them to solve problems. Davis argued that it is better to *begin* with problems, allow students to develop methods for solving them, and recognize that what students take away from this experience is what they have learned. Such learning is likely to be deep and lasting. Davis referred to the learning that students take with them from solving problems as "residue."

The point that both Brownell and Davis were making is that we build understandings or relationships by discovering them and hearing about them and using them as we solve problems. Teachers can point out relationships, but they become meaningful as students use them for solving problems. For example, teachers can point out that 38 means 38 ones, or 3 tens and 8 ones, or 2 tens and 18 ones, and so on. But these relationships only become meaningful for students when they use them to solve problems. For example, if students solve 45 + 38 by adding 3 tens and 4 tens to get 70, and 8 ones and 5 ones to get 13 ones, and then combine these to get 83, then the relationships between tens and ones become significant.

Thinking of understandings as outcomes of solving problems rather than as concepts that we teach directly requires a fundamental change in our perceptions of teaching. Many of us have been brought up to think that the best way to teach mathematics is to teach important concepts, like place value or common denominators, by explaining them clearly and demonstrating how to use them and then having students practice them. Our recommendation is that we change our way of thinking and teaching so that students are allowed to develop concepts, such as place value and common denominators, in the context of solving problems. This means that when selecting tasks or problems, we need to think ahead about the kinds of relationships that students might take with them from the experience.

during the weeks leading up to this event. On this day, they set up 260 extra seats for the game and charged $5000 for each seat, with the proceeds used to help find treatments and a cure for Lou Gehrig's disease. There are many questions that could be asked about this situation, including statistical comparisons of Mr. Gehrig's and Mr. Ripken's baseball careers, estimates on the number of people that have seen, in person, each of them play, and percentage of revenue from today's game that was contributed to fight Lou Gehrig's disease. Of course, to answer the questions students would need to do some additional research. Tasks like this provide experiences in finding, organizing, and manipulating lots of information. Students are likely to take with them a variety of specific techniques for organizing and manipulating numbers.

But it is likely that students will take with them another kind of strategy from solving both kinds of problems that is even more important than the specific techniques they acquire. As students develop their own methods for solving problems, they develop general approaches for inventing specific procedures or adapting ones they already know to fit new problems. In other words, they learn how to construct their own methods (Fennema et al. 1993; Hiebert and Wearne 1993; Kamii and Joseph 1989; Wearne and Hiebert 1989).

This kind of residue is extremely valuable because it enables students to solve a variety of problems without having to memorize different procedures for each new problem. Although students can acquire specific strategies for specific tasks through more traditional forms of instruction, we believe that they acquire general approaches for developing their own procedures only if they are allowed to treat tasks as problematic. In other words, students learn how to construct methods to solve problems if they are allowed to do just that.

A major advantage of thinking about learning as the residue that gets left behind when solving problems is that it provides a way of dealing with a very common difficulty. Many students have trouble connecting the concepts they are learning with the procedures they are practicing (Hiebert 1986). They often end up memorizing and practicing procedures that they do not understand (e.g., adding fractions with unlike denominators). This has damaging consequences, such as forgetting procedures, learning slightly flawed procedures without knowing it, or applying them rigidly without adjusting them for slightly different problems (National Assessment of Educational Progress 1983). In general, if students separate their conceptual understandings from their procedures it means that they cannot solve problems very well.

We believe that the reason so many students separate concepts and procedures, and acquire many procedures they do not understand, is that traditional instruction encourages this separation. By trying to teach concepts and procedures directly, we artificially separate them. Although we may try to get students to hook them back together, this is more difficult than we think and most students are not successful. They learn procedures by imitating and practicing rather than by understanding them, and it is hard to go back and try to understand a procedure after you have practiced it many times (Hatano 1988; Resnick and Omanson 1986; Wearne and Hiebert 1988a). Without understanding, it is easy to forget procedures and distort them. And it is hard to adjust them to solve different kinds of problems.

An alternative is to begin with problems. If students are encouraged to develop their own procedures for solving problems, then they must use what they already know, including the understandings they have already constructed. There is no other way to do it. Understandings and procedures remain tightly connected because procedures are built on understandings. The methods students first develop may not be the most efficient ones, but they will be methods students understand. This is exactly what we are finding in classrooms that treat arithmetic in this way (Carpenter et al. 1989; Hiebert and Wearne 1992, 1993, in press; Kamii and Joseph 1989; Murray et al. 1992).

We believe that if we want students to understand mathematics, it is more helpful to think of understanding as something that results from solving problems, rather than something we can teach directly. In particular, we believe that teaching concepts and procedures separately is potentially damaging. It is more appropriate to engage students in solving problems because it is only through solving problems that their concepts and procedures develop together and remain connected in a natural and productive way.

What Changes Should We Make in Our Current Curriculum?

In this chapter we have described the kinds of tasks that fit into the system of instruction we outlined in Chapter 1. Most of our discussion says more about *how* the content should be treated than *what* content should be included. The system of instruction that we recommend is an approach to treating content, not a prescription for the selection of content.

Nevertheless, we can say a few things about content: First, we believe that much of the content in current curricula, as presented in popular textbooks, is appropriate *as long as* students are allowed to make the mathematics problematic. The system of instruction we describe does not mean a wholesale replacement of the curriculum. In fact, there may be few content changes that are required.

A second point is that the reason for including particular topics may be different now than in the past. Arithmetic computation, for example, has occupied the lion's share of the curriculum in elementary school because of the importance that has been attached to rapid paper-and-pencil calculation skills. This is being challenged by the reform documents which point out that these skills are rapidly declining in importance (NCTM 1989). We agree that students do not need to become high-speed paper-and-pencil calculators; electronic calculators do that job better. The great amount of time spent practicing fast execution of paper-and-pencil procedures is better spent elsewhere. But we believe that computation is still an important topic (Hiebert 1990). It provides a rich site for students to develop methods for solving problems and to gain important understandings about the number systems and about operations within number systems. Studying computation serves as a vehicle for building mathematical understandings. Of course, it still is useful to possess some computation skills, but these develop alongside the insights into how numbers work as students develop their own methods and examine them carefully (Carpenter et al. 1989; Fennema et al. 1996; Hiebert and Wearne 1992, 1993, in press).

A third point about content is that the criteria identified earlier can be used to decide whether classroom tasks contain appropriate content. The task should allow and encourage students to problematize the mathematics of the situation, and it should invite students to use the tools they already possess to solve the problem. Such tasks are likely to leave behind something of mathematical value. These criteria cannot be used to select topics or to say that one topic is more important than another. But they do say that tasks should be selected for the mathematics of the situation, rather than other extraneous features and that, as one completes the task and looks back, the mathematics of the situation should be the most salient residue. Mathematics should be the focal point, both going into the task and coming out of the task.

Using these three criteria, it is easy to see that much of the content in the current curricula could be framed into tasks that would be appropriate. On the other hand, some tasks that are being proposed as innovative and reform-minded would be inappropriate. Simple computation problems, such as 38 + 45 and $\frac{1}{4} + \frac{1}{3}$ can be mathematically problematic for students if they are introduced at the right time and treated appropriately, and they can leave behind important residue. In contrast, planning parties with $100 budgets might look interesting and engaging, but might have few mathematical goals going in and leave little mathematical residue coming out. When deciding whether a task is appropriate, it is helpful to look at the way in the which the goals students set will shape the task and the kind of mathematical understandings that are likely to be left behind.

Tasks Form the Foundation for Instruction

The system of instruction we outlined in Chapter 1 is an interrelated ensemble of five dimensions. Instruction depends on all five working together, and the nature of the tasks is only one of the five. Still, the tasks provide a foundation for instruction that is critical. The underlying processes of reflection and communication are possible only when the tasks are appropriately problematic. The entire system of instruction we are describing depends on tasks that allow and encourage students to treat mathematics as problematic. The way in which the other classroom dimensions build on these kinds of tasks will become clear in the next several chapters.